B.TECH. TCS-702

SEVENTH SEMESTER EXAMINATION, 2007-2008

DIGITAL IMAGE PROCESSING

Time : 3 Hours *Total Marks : 100*

Note. (1) All questions are compulsory.
(2) All questions carry equal marks.

Q.1. Attempt any four questions of the following: (5×4=20)

Q.1. (*a*) Consider the two image subsets S_1 and S_2:

For $V = \{1\}$, determine whether S_1 and S_2 are

4-connected

8-connected

m-connected

	S_1				S_2				
1	1	1	0	0	1	1	1	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	1	1	0	1	0	0	1
1	0	0	1	0	0	1	0	0	1
0	1	1	0	0	0	0	0	0	0

Ans. We have two image subsets S_1 and S_2

$V = \{1\}$

	S_1				S_2				
1	1	1	0	0	1	1	1	0	0
0	0	0	0	0	(1) Q	1	1	0	0
0	0	0	1	P (1)	0	(1)	0	0	1
1	0	0	(1)	0	0	1	0	0	1
0	1	1	0	0	0	0	0	0	0

4-Connected: Let us consider two points P and Q. P and Q are not 4-connected as they do not belong to 4-neighbours of each other.

8-connected: These two pixels are 8-connected as the Q belongs to the 8-neighbours of P.

$$Q \in N_8(P)$$

***m*-connected:** These two pixels are *m*-connected as $Q \in N_D(P)$ and $N_4(P) \cap N_4(Q)$ has no pixels whose values are from V.

(*b*) A common measure for the transmission of digital data is baud rate. Generally, transmission is accomplished in packets consisting of a start bit, a byte (8 bits) of information, and a stop bit. Using these facts answer the following:

(1) How many minutes would it take to transmit 1024 × 1024 image with 256 gray levels using a 56 K baud modem?

(2) What would the time be if the same image is transmitted over a 750 K baud phone connection?

Ans. (1) We have 256 gray levels, it means

$256 = 2^8$

∴ 8 bits for information

1 bit for start

1 bit for stop.

Total bits = 10

Total number of bits required

$$= 1024 \times 1024 \times 10$$
$$= 2^{10} \times 2^{10} \times 10$$
$$= 2^{20} \times 10$$

$$\text{Time required} = \frac{2^{20} \times 10}{56 \times 1000}$$

$$= 187.3 \text{ sec} = 3.1 \text{ min}$$

(2) Time required $= \frac{2^{20} \times 10}{750 \times 1000}$

$= 13.98$ sec

(*c*) **The 4×4 input image is defined by the following matrix with gray scale [0.9]:**

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

Draw the image histogram and show the new output image along with its histogram after histogram equalization.

(*d*) **The following matrix defines a 5×5 image $f(x, y)$. The center pixel $f(2, 2)$ is underlined. Suppose smoothing is done to the image using 3×3 neighborhood in the spatial domain. Then what will be the new value $f(2, 2)$ using the**

(1) the mean filter

(2) weighted average filter

(3) median filter

(4) min filter and

(5) max filter

0	1	0	6	7
2	0	1	6	5
1	1	7	5	6
1	0	6	6	5
2	5	6	7	6

Ans. We have, 5×5 image matrix as follows:

0	1	0	6	7
2	0	1	6	5
1	1	[7]	5	6
1	0	6	6	5
2	5	6	7	6

Square value is $f(2, 2)$ this will be centre pixels. Let us suppose the mask of size 3×3 is

1	1	1
1	1	1
1	1	1

By applying mask on centre pixel.

The value of centre pixel $f(2, 2)$ is 32.

1. Mean filter: By applying average mean filter

$$f(2, 2) = \frac{1}{mn}[\Sigma g(s,t)]$$

$$= \frac{1}{9} \times 32 = 3.55 = 4$$

2. Weighted average filter

$$f(2, 2) = \frac{32}{9} = 4$$

3. Median filter $f(2, 2) = 32$.

4. Min filter $f(2, 2) = 0$.

5. Max filter $f(2, 2) = 7$.

(*e*) **Give the Roberts cross gradient and Sobel opertors. How these masks can be used to implement spatial domain filtering?**

Ans. Roberts cross gradient operators

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

For Roberts the mask will be

For x

−1	0
0	1

For y

0	−1
1	0

Operators will be

$$G_x = z_9 - z_5$$
$$G_y = z_8 - z_6$$

Sobel operators: The mask will be

For x

−1	−2	−1
0	0	0
1	2	1

For y

−1	0	1
−2	0	2
−1	0	1

Operators:

$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$
$$G_7 = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

The sobel and roberts operators are used to preserve the isotropic feature property. The mask used to approximate the gradient gives the same result only for vertical and horizontal edges and thus the isotropic properties of the gradient are preserved only for multiples of 90°.

2. Atempt any two of the following: (10×2=20)

2. (a) The basic approach used to approximate a discrete deriative (as in spatial domain) involves taking difference of the form

$$f(x+1, y) - f(x, y)$$

(1) Obtain a filter transfer function, $H(u, v)$ for performing the equation process in the frequency domain.

(2) Show that $H(u, v)$ is a highpass filter.

Ans. $f(x + 1, y) - f(x, y)$

$$f(x, y) = \sum_{u=0}^{n-1} \sum_{v=0}^{n-1} (u, v) h(x, y, u, v)$$

$$F = \sum_{u=0}^{n-1} \sum_{v=0}^{n-1}$$

$$F(x, y) = g(x, y) - \omega(x,y)\, \eta(x, y)$$

$$\sigma^2(x, y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=a}^{a} \sum_{t=-b}^{b} [\hat{f}(x+s, y+t) - \bar{\hat{f}}(x, y)]^2$$

$$\bar{\hat{f}}(x, y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=b}^{b}$$

$$\hat{f}(x+s, y+t)$$

$$f(x + 1, y) - f(x, y)$$

(1) Filtering in Frequency Domain: Frequency domain in nothing more than the space defined by values of the fourier transform and its frequency variables (u, v)

$$H(u, v) = \begin{cases} 0 & \text{if } (u, v) = (M/2, N/2) \\ 1 & \text{otherwise} \end{cases}$$

All this filter would do is set $F(0, 0)$ to zero and leave all other frequency component of the fourier transform untouched as desired.

$$f(x, y) * h(x, y)$$

$$= \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n) h(x-m, y-n)$$

$$f(x, y) * h(x, y) \Leftrightarrow F(u, v) H(u, v)$$
$$f(x, y)\, h(x, y) \Leftrightarrow F(u, v) * H(u, v)$$
$$f(x, y) * h(x, y) \Leftrightarrow F(u, v) H(u, v)$$
$$\delta(x, y) * h(x\, y) \Leftrightarrow \}[\delta(x, y)] H(u, v)$$
$$h(x, y) \Leftrightarrow H(u, v)$$

(2) $H(u, v)$ is a highpass

$$H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \le D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$$

$$H(u, v) = \frac{1}{1 + [D_0 / D(u, v)]^n}$$

$$H(u, v) = 1 - e^{D^2(u, v)} / 2D_0^2$$

It can be shown that

$$\Im\left[\frac{d^n f(x)}{dx^n}\right] = (ju)^n F(u)$$

$$\Im\left[\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right]$$

$$= (ju)^2 F(u, v) + (jv)^2 F(u, v)$$
$$= -(u^2 + v^2)\, F(u, v)$$

$$\Im[\nabla^2 f(x, y)] = -(u^2 + v^2) F(u, v)$$

$$H(u, v) = -(u^2 + v^2)$$

This is the high pass filter.

2. (b) Given a matrix of size 3 × 3 as

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 2 \end{pmatrix}$$

Compute |A|, A^{-1}, Trace of A, Eucledian norm of A, Eigen values and Eigen vectors of A.

Ans. Eigen value

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\varepsilon_x = \frac{1}{16}\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$= \frac{1}{16}\begin{bmatrix} 3 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & 3 & -1 \end{bmatrix}$$

$$C_y = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ 0 & & \ddots & \\ & & & \lambda_n \end{bmatrix}$$

$$|A|^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\text{Eigen vector} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -1 \\ -3 & 4 & 2 \end{bmatrix}$$

2. (c) What is meant by singularity and ill-condition in relation to image restoration? Derive expression of restored image using least-square approach. Comment on the singularity of this filter.

Ans. We are interested generally by the combination of an illumination source and the reflection or absorption of energy from that source by the elements of the sense being imaged. We enclose illumination and scene in quotes to emphasize that fact that they are considerably more general than imaged

$$\phi(x, y) = \phi(x), \phi(y)$$
$$\psi^H(x, y) = \psi(x)\ \psi(y)$$
$$\psi^V(x, y) = \psi(x)\ \psi(y)$$
$$\psi^D(x, y) = \psi(x)\ \psi(y)$$
$$f(x, y)$$

$$= \frac{1}{\sqrt{MN}}\sum_m \sum_n W_\phi(j_0, m, n)\psi j_o mn(x, y)$$

$$+ \frac{1}{\sqrt{MN}} \sum_{i = HV} \sum_{D = j = j_o}^{\infty}$$

$$\sum_m \sum_n W^i \psi(j, m, n) \psi^j m, n(x, y)$$

3. Attempt any two of the following: (10×2=20)

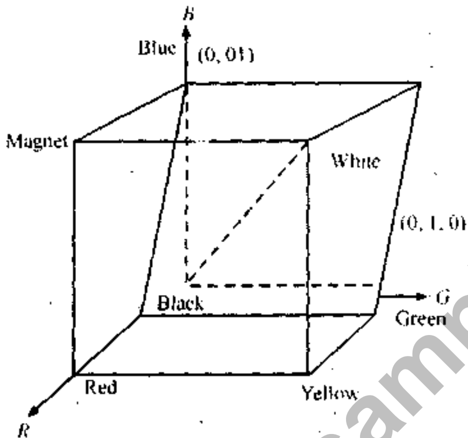
3. (a) Let an RGB image is given as an input:

(1) Convert the image into monochrome (linear and non-

linear), normalized RGB and HSI image.

(2) Suppose RGB color triplet for a particular color is given by (0.3, 0.5, 0.2). Compute corresponding YIQ and HSV triplets.

Ans. (1) RGB Color model: This mode is based on a cartesian coordinate system. The color subspace of interest is cube shown in fig.



(2) Suppose RGB color triplet for a particular color is given by

(0.3, 0.5, 0.2) compute.

Corresponding *YIQ* and *HSV* triplets.

This subset colors is called the set of safe RGB color or the set cell systems safe colors. In internet application they are called safe web colors or safe browser colors.

3. (*b*) Suppose two discrete one dimensional functions are represented by the sequences

f = [5 7 11 8 2 6 8 9 7 4 3]

h = [1 2 1]

Computer $f \oplus h$, $f \ominus h$, $f \circ h$ and $f \cdot h$. Also plot the corresponding graphs.

Ans. Discrete and dimensional function: Dscrete one dimensional function are represented $N \times N$ whose forward discrete transform $T(u, v)$ can be expressed in terms of general relation

$$T(u, v) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) g(x, y, u, v)$$

Suppose

$v = 0, 1, 2,\ \ N = 1$ given $T(u, v) f(x, y)$

$$f(x, y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} T(u, v)\, h(x, y, u, v)$$

$x, y = 0, 1, 2 \ldots N-1$. In the equation $g(x, y, u, v)$ and $h(x, y, u, v)$ are sequence.

3. (*c*) Suppose that *A*, *B* and *C* are three points. Prove that

(1) $(((A \cdot B) \circ C) \cdot B) \circ C = (A \cdot B) \circ C$

(2) $(((A \circ B) \cdot C) \circ B) \cdot C = (A \circ B) \cdot C$

Ans. (1) $(((A \cdot B) \circ C) \cdot B) \circ C = (A \cdot B) \circ C$

$$(A \ominus B)^C = A^C \oplus \hat{B}$$

$$(A \ominus B)^C = \{Z \mid (B)_Z \subseteq A\}^C$$

$$(A \ominus)^C = \{Z \mid B)_Z \cap A^C = \phi\}^C$$

Z's satisfy $(B)_Z \cap A^C = \phi$

$$(A \ominus B)^C = \{Z\, (B)_Z \cap A^C \neq \phi\}$$

$$= A^C \oplus \hat{B} = (A \cdot B) \circ C \quad \textbf{Ans.}$$

(*ii*) $(((A \circ B) \cdot C) \circ B) \cdot C = (A \circ B) \cdot C$

$$A \cdot B = (A \ominus B) \oplus B$$

$$A \cdot B = (A \oplus B) \ominus B$$

$$A \cdot B = U\{(B)_Z \mid (B_Z \subseteq A)\}$$

$$A \cdot B = U\{(B)_Z \mid (B)_Z \subseteq A\}$$

$$(A \cdot B)^C = (A^C \cdot \hat{B}) = (A \circ B) \cdot C \quad \textbf{Ans.}$$

4. Attempt any two of the following: (10×2=20)

4. (*a*) Suppose an image contains two types of regions, R_1 and R_2. The priori probability that a pixel belongs to R_1 is 0.4 and to R_2 is 0.6. Probability density function of intensity in R_1 and R_2 are denoted by $p_1(z)$ and $p_2(z)$ respectively, where

$p_1(z) = 0.2 - 0.04|5 - z|$ for $0 \le z \le 10$

$p_2(z) = 0.2 - 0.04|10 - z|$ for $5 \le z \le 15$

Determine the optimum threshold for image segmentatin by the gray level thresholding technique.

Ans. $P_1(z) = \begin{cases} ae^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$

$$\mu = \frac{1}{a}$$

$$\sigma^2 = \frac{1}{a^2}$$

$$P_2(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}$$

and its variance by

$$\sigma^2 = \frac{(b^2-a)^2}{12}$$

$$P_z = \begin{cases} P_1 & \text{for } z = a \\ P_2 & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

4. (b) Write short notes on:

1. Region Merging and Region Splitting.

2. Watershed Segmentation Algorithm.

Ans. (1) Region Merging and Region Splitting: An alternate is to subdivide into of arbitary disjointed regions and then merge and/or split the region in an attempt to satisfy the conditions stated. A split and merge algorithm that relatively works forwards satisfying these constraints is developed.

1. Split into four disjoint quadrants any region R_i for which $P(R_i)$.

2. Merge any adjacent region R_i & R_k for which $P(R_j \cup R_k)$.

3. Stop when no further merging or splitting is possible.

2. Watershed Segmentation Algorithm: We have discussed segmentation based on three principal concept

1. thresholding

2. deletion of discontinuities

3. region processing

Based on the concept of so called morophological watersheds.

4. (c) How many degrees of freedom are there in a plane projective transformation? Name the properties that are preserved under such transformations. What simplification needs to be imposed on plane projective transformation to arrive at plane affine transformation? Give the physical interpretation of parameters of plane affine transformation.

Ans. The principal advantage of piecewise linear function over the types of functions. We have discussed thus for is that the form of piecewise function can be orbitarily complex. In facts, as well see shortly a practical implementation of same important transformations can be formulated only as piecewise function physical parameters of plane affine transformation.

5. Attempt any two of the following:

(10×2=20)

5. (a) Suppose in a two-class pattern recognition problem, classes are distributed as Gaussian where mean vectors and convariance matrices are as follows:

For class-I: $\mu 1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \Sigma 1 = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

and

For class-II: $\mu 2 = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, $\Sigma 1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

Determine the class boundary considering Bayesian classification scheme. Assume a priori probabilities of class-I and class-II are 0.4 and 0.6 respectively.

Ans.

$$F(u, v) = \frac{1}{M}\sum_{x=0}^{M-1} e^{-j2\pi ux/M}$$

$$\frac{1}{M}\sum_{y=0}^{N-1} f(x, y)e^{-j2\pi uy/N}$$

$$\mu_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

For class 1 $\quad = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

For class 2 $\quad \mu_2 = \begin{pmatrix} 5 \\ 7 \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$$= \boxed{f(x, y)} \rightarrow \boxed{F(x, y)} \rightarrow \boxed{F(u, v)}$$

$$= \begin{pmatrix} 5 \\ 7 \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

For class 1 $= \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 3 \end{pmatrix}$

For class 2 $= \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

5. (*b*) Obtain the gray-level co-occurrence matrix of a 5×5 image composed of a checkerboard of alternating 1's and 0's. The position operator *P* is defined as "one pixel to the right". Assume that the top level pixel has value 0.

Ans. One Pixel to the right: A pixel p at coordinates (x, y) has four horizontal and vertical neighbours whose coordinates are given by

$(x + 1, y), (x - 1, y), (x, y + 1), (x, y - 1)$

This set of pixels, called the 4-neighbors of P is denoted by $N_4(P)$. Each pixel is a unit distance from (x, y) and same of the neighbors of P lie outside the digital image if (x, y) is on the border of the image. The four diagonal neighbors of P have coordinates

$(x + 1, y + 1), (x + 1, y - 1), (x - 1, y + 1) (x - 1, y - 1)$

5. (c) Explain any two regional descriptors in short. Given a 4*4 image whose gray levels ordered lexicographically are as follows:

2 3 0 1 1 3 1 2 0 2 0 3 1 1 2 3

Calculate the spatial moments up to the second order.

Ans. k-point boundary in the *xy*-plane starting at an orbitary point (x_0, y_0) coordinate pairs (x_0, y_0) (x_1, y_1) (x_2, y_2) ... (x_{k-1}, y_{k-1}) are encountered in transversing the boundary

$S(k) = [S(k), y(k)]$ for

$k = 0, 1, 2 \ldots k - 1$

$S(k) = x(k) + jy(k)$

2 3 0 1 1 3 1 2 0 2 0 3 1 1 2 3

Chain code: 0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1

Difference: 3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0

Shape no: 0 0 0 3 1 0 3 3 3 0 1 3 0 0 3 1 3 0 3

Spatial moment upto second order.