B.TECH.

EME-202

SECOND SEMESTER EXAMINATION 2009-10

ENGINEERING MECHANICS

Time : 3 Hours *Total Marks : 100*

Note: (i) This paper is in three sections. Section A carries 20 marks, Section B carries 30 marks and Section C carries 50 marks.

(ii) Attempt all questions. Marks are indicated against each question part.

(iii) Assume missing data suitably, if any.

Section-A

Q.1. You are required to answer all the parts:

(10×2=20)

Choose correct answer for the following:

Q.1. (*a*) The necessary and sufficient condition for a system of coplanar forces to be in equilibrium:

(*i*) $\Sigma F_x = 0$

(*ii*) $\Sigma F_x = \Sigma F_y = 0$

(*iii*) $\Sigma M_0 = 0$

(*iv*) $\Sigma F_x = \Sigma F_y = \Sigma M_0 = 0$

Ans. (*iv*) $\Sigma F_x = \Sigma F_y = \Sigma M_0 = 0$

Q.1. (*b*) The bending equation is:

(*i*) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

(*ii*) $\frac{M}{y} = \frac{\sigma}{I} = \frac{E}{R}$

(*iii*) $\frac{M}{y} = \frac{\sigma}{R} = \frac{E}{I}$

(*iv*) $\frac{M}{I} = \frac{\sigma}{R} = \frac{E}{y}$

Ans. (*i*) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

Q.1. (*c*) The principle of conservation of energy can't be applied in the following situation:

(*i*) body sliding down on a rough inclined plane.

(*ii*) simple pendulum

(*iii*) a particle executing SHM

(*iv*) a particle moving in a gravitational field

Ans. (*i*) body sliding down on a rough inclined plane.

Q.1. (*d*) In UDL loading (*w* N/m), the maximum bending moment in case of simple supported beam is given as:

(*i*) wL (*ii*) $wL^2/2$

(*iii*) $wL^2/4$ (*iv*) $wL^2/8$

Ans. (*iv*) $wL^2/8$

Fill in the blanks for the following parts:

You will be awarded full marks, if all the entries in a part are correct otherwise will be awarded zero.

Q.1. (*e*) The algebric sum of the moments of two ______ forces with respect to any moment centre in their plane of action is equal to the moment of their ______ with respect to the same centre.

Ans. Concurrent or non concurrent or parallel force or coplaner, Resultant

Q.1. (*f*) In a cantilever beam carrying a concentrated load at the free end, the bending moment will be zero at ______ and maximum at ______.

Ans. Free end, Fixed End.

Q.1. (*g*) **The angular velocity (rad/sec) of a body rotating at N rpm is ______ and the linear velocity of a body rotating at ω rad/sec along a circular path of radius *r* is _______.**

Ans. $\omega = \frac{2\pi N}{60}$, $V = r\omega$.

Q.1. (*h*) **In truss analysis, all forces acting on truss are applied at the ______ only and also lie in the _______ of truss.**

Ans. Joint, Plane

Match the columns for the following partrs: You will be awarded full marks, if all the matches in a part are correct otherwise will be awarded zero.

Q.1. (*i*) **Match the following columns. Column II shows the moment of inertia about a centroidal axis:**

Column I	Column II
(*i*) Triangle	(P) $0.11\ R^4$
(*ii*) Circle	(Q) $\pi R^4/4$
(*iii*) Semicircle	(R) $bh^3/12$
(*iv*) Rectangle	(S) $bh^3/36$

Ans. (*i*) Triangle → $bh^3/36$

(*ii*) Circle → $\pi R^4/4$

(iii) Semicircle → $0.11\ R^4$

(Iv) Rectangle → $bh^3/12$

Q.1. (*j*) **Match the following columns.**

Column I	Column II
(*i*) Curvilinear motion	(P) Neither pure rotation nor pure translation
(*ii*) Rectiliaear motion	(Q) pure rotary motion
(*iii*) General plane motion	(R) Motion of particles remains parallel & straight
(*iv*) Instantaneous motion	(S) Motion of particles remains parallel and in curve

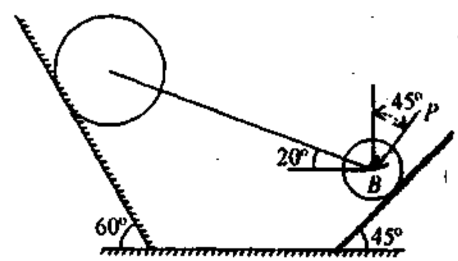
Ans.

(*i*) Curvilinear motion	(S) Motion of particles remains parallel and in curve
(*ii*) Rectilinear	(R) Motion of particles remains parallel & straight
(*iii*) General plane motion	(R) Neither pure rotation nor pure translation
(*iv*) Instantaneous motion	(Q) pure rotary motion

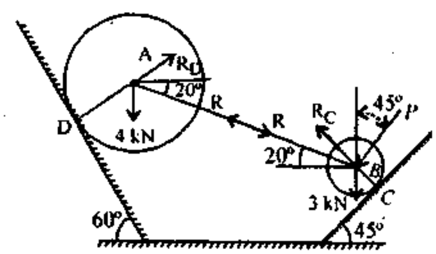
SECTION B

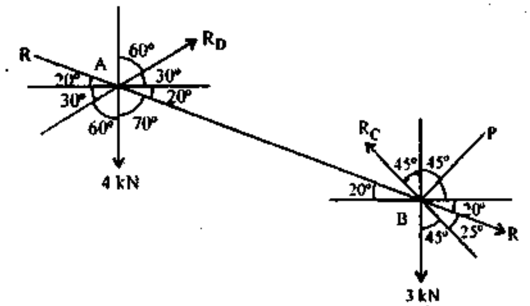
Q.2. Answer any three parts of the following:

(a) Two cylinders *A* and *B* weighing 4 kN and 3 kN, respectively, rest on smooth inclined plane as shown in figure. They are connected by a bar of negligible weight hinged to each cylinder at its geometric centre by smooth pins. Find the force *P* to be applied to the smaller cylinder at 45° to the vertical to hold the system in the given position.



Ans. Free body diagram is shown below:





Applying Lami's Theorem at '*A*'

$$\frac{4000}{\sin 130°} = \frac{R}{\sin 120°} = \frac{R_D}{\sin 110°}$$

$$R = 4522 \text{ N}$$

$$R_D = 4000 \times \frac{\sin 110°}{\sin 130°} = 4906.7 \text{ N}$$

For equilibrium of point *B*, we have.

$\Sigma F_x = 0$

$R \cos 20° - R_C \cos 45° - P \cos 45° = 0$

$4522 \cos 20° - (R_C + P) \cos 45° = 0$

or $R_C + P = 6010.31$ ...(1)

$\Sigma F_y = 0$

$R_C \cos 45° - 3000 - R \cos 70° - P \cos 45° = 0$

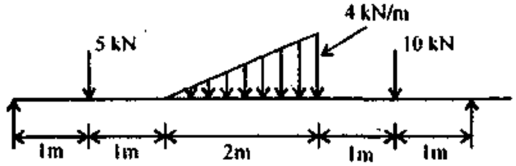
or $R_C - P = 6430.85$ ...(2)

Adding (1) and (2), we get

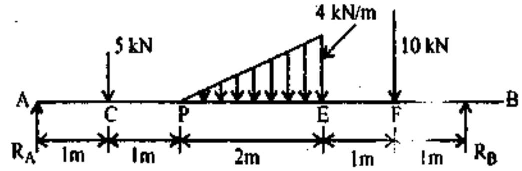
$R_C = 6220.58$ N

$P = 210.27$ N **Ans.**

(b) Calculate the values of shear force and bending moments for the simple supported beam shown in figure. Also draw the shear force and bending moment diagrams.



Ans. Let R_A and R_B are support reactions at points *A* and *B*. F.B.D. is shown below:



For the equilibrium $\Sigma F_y = 0$

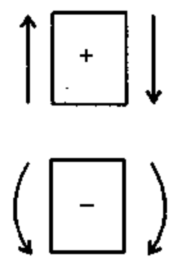
$$R_A + R_B = 5 + 10 + \frac{1}{2} \times 2 \times 4$$

$R_A + R_B = 19$ kN ...(1)

$\Sigma m_A = 0$

$$R_B \times 6 = 5 \times 1 + 10 \times 5 + \frac{1}{2} \times 2 \times 4\left(2 + \frac{2}{3} \times 2\right)$$

$$R_B \times 6 = 5 + 50 + 4 \times \frac{10}{3}$$



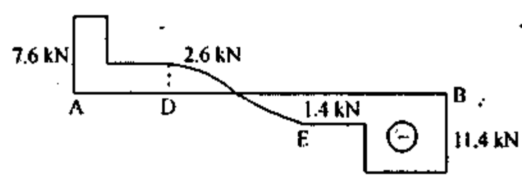
$R_B \times 6 = 68.3$

$R_B = 11.4$ kN

$R_A = 7.6$ kN

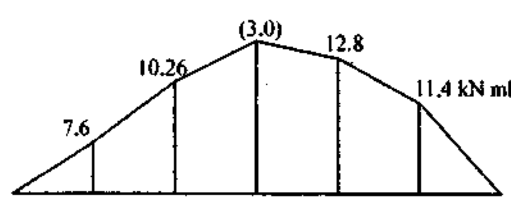
Distance = 3.61 m from point '*A*' maximum *Bm* = 13.03 kNw

Shear force diagram is shown below:



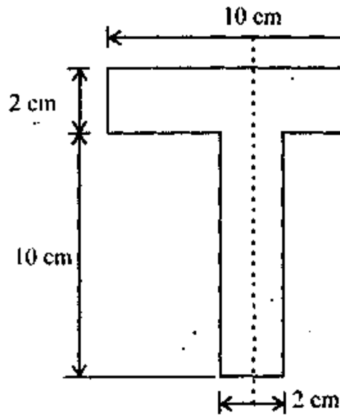
SFD

Bending moment diagram is shown below:



BMD

(c) Determine the moment of inertia of T section about the horizontal and vertical axes, passing through the C.G. of the section as shown figure.

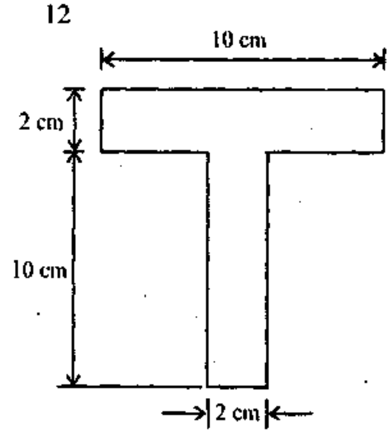


Ans. Give T section is symmetrical about y-axis

$$I_{yy} = \frac{2\times10^3}{12} + \frac{10\times2^3}{12} = \frac{2000}{12} + \frac{80}{12} = \frac{2080}{12}$$

$$I_{yy} = 173.3333 \text{ cm}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$



Here $A_1 = 20 \quad y_1 = 5$

$A_2 = 20 \quad y_2 = 11$

$$\bar{y} = \frac{20\times5 + 20\times11}{20+20} = \frac{20(5+11)}{20(2)}$$

$$\bar{y} = 8 \text{ cm}$$

$$I_{xx} = I_{x_1x_1} + A_1(y_1 - \bar{y})^2 + I_{x_2x_2} + A_2(y_2 - \bar{y})^2$$

$$= \frac{2\times10^3}{12} + 20(5-8)^2 + \frac{10\times2^3}{12} + 20(11-8)^2$$

$$= \frac{2000}{12} + 180 + \frac{80}{12} + 180 = \frac{2080}{12} + 360 = \frac{2080+4320}{12}$$

$$I_{xx} = 533.3333 \text{ cm}$$

$$I_{yy} = \frac{1}{12}\times10\times(2)^3 + \frac{1}{12}\times2\times10^3 = \frac{1}{12}[80 + 2000]$$

$$I_{yy} = 173.34 \text{ cm}^4.$$

(d) A solid shaft is subjected to a maximum torque of 15 MN-cm. Determine the diameter of the shaft. if the allowable shear stress and the twist are limited to 1 kN/cm^2 and 1°, respectively for 210 cm length of shaft. G = 8 MN/cm^2.

Ans. Maximum torque $T = 15$ MN-cm

Allowable stress = 1 kN/cm^2

$$\text{Twist} = 1° = \left(\frac{\pi}{180}\right) \text{Rad.}$$

Length = 210 cm

$G = 8$ MN/cm^2

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

$$\tau = \frac{T}{J} \times r$$

$$1 \times 10^3 = \frac{15 \times 10^6}{\frac{\pi}{32} d^4} \times \frac{d}{2}$$

$$\frac{1 \times 10^3 \times 2\pi}{15 \times 10^6 \times 32} = \frac{1}{d^3} \quad \Rightarrow \quad d^3 = \frac{15 \times 32 \times 10^6}{1 \times 10^3 \times 2\pi}$$

$$d = 42.4 \text{ cm} \qquad \text{...(1)}$$

$$\theta = \frac{TL}{GJ}$$

[Larger value of diameter is safe]

[So $d = 42.4$ cm]

$$\frac{\pi}{180} = \frac{15 \times 10^6 \times 210}{8 \times 10^6 \times \pi d^4 / 32}$$

$$d^4 = \frac{15 \times 210 \times 180 \times 32}{8 \times 9.86} \quad \Rightarrow \quad d = 2189 \text{ cm} \qquad \text{...(2)}$$

From (1) and (2), larger value of diameter is 42.4 cm. Hence it will be selected.

(e) The motion of particle is given by $a = t^3 - 3t^2 + 5$, where a is the acceleration in m/sec^2 and t is the time in seconds. The velocity of the particle at $t = 1$ sec is 6.25 m/sec, and the displacement is 8.30 meters. Calculate the displacement and the velocity at $t = 2$ sec.

Ans. Given $a = t^3 - 3t^2 + 5$

or
$$\frac{dV}{dt} = a = t^3 - 3t^2 + 5 \qquad \text{...(1)}$$

Integrating on both sides

$$V = \frac{t^4}{4} - \frac{3t^3}{3} + 5t + C_1 = \frac{t_4}{4} - t^3 + 5t + C_1 \qquad \text{...(2)}$$

When $t = 1$ sec $V = 6.25$ m/s

$$625 = \frac{1}{4} - 1 + 5 + C_1$$

$C_1 = 2$, So velocity at $t = 2$ sec ...(3)

$$V = \frac{2^4}{4} - 2^3 + 5 \times 2 + 2$$

$V = 8$ m/s **Ans.**

Displacement at $t = 2$ seconds,

Again $$\frac{dS}{dt} = \frac{t^4}{4} - t^3 + 5t + 2$$

$$dS = \left(\frac{t^4}{4} - t^3 + 5t + 2\right)dt$$

Integrating $$S = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + C_2$$

at $t = 1$ sec, $S = 8.30$ m

$$8.30 = \frac{1}{20} - \frac{1}{4} + \frac{5}{2} + 2 + C_2$$

or $$8.30 = 4.3 + C_2$$

$$C_2 = 4$$

Substituting the value of C_2, we get

$$S = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + 4$$

Now, displacement at $t = 2$ sec is

$$S = \frac{32}{20} - \frac{16}{4} + \frac{20}{2} + 4 + 4 \quad \text{or} \quad S = 15.6 \text{ m.}$$

SECTION - C

Q.3. Answer any TWO parts of the following: **(2×5=10)**

(a) State and prove Varignon's theorem.

Ans. Varignon's theorem: It states that the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.

Principle of moment states that the moment of the resultant of a number of faces about any point is equal to the algebraic sum of moments of all the forces of the system about the same point.

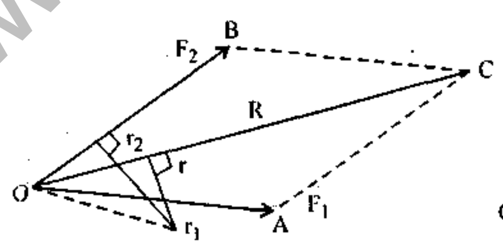


Fig. (a)

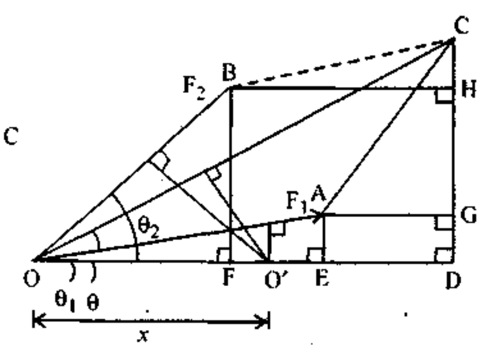


Fig. (b)

Let

θ_1 = Angle made by F_1 with OD

θ = Angle made by R with OD

θ_2 = Angle made by F_2 with OD

r_1 = ⊥ distance between F_1 and O'

r = ⊥ distance between R and O'

r_2 = + distance between F_2 and O'

According to Varignon's principle

$$R \times r = F_1 \times r_1 + F_2 \times r_2$$

from Fig. (*b*)

$$F_1 \sin\theta_1 = AE = GD = CH$$

$$F_1 \cos\theta_1 = OE$$

$$F_2 \sin\theta_1 = BF = HD$$

$$F_2 \cos\theta_2 = OF = ED \qquad \because OB = AC \text{ and also } OB \parallel AC \text{ (hence } OF = ED)$$

$$R \sin\theta = CD$$

$$R \cos\theta = OD$$

Let the length $OO = x$

Then $x \sin\theta_1 = r_1$, $x \sin\theta = r$, $x \sin\theta_2 = r_2$

Now moment of R about O'

$$= R \times r$$

$$= R \times x \sin\theta$$

$$= (R \sin\theta) \times x$$

$$= CD \times x$$

$$= (CH + HD) \times x$$

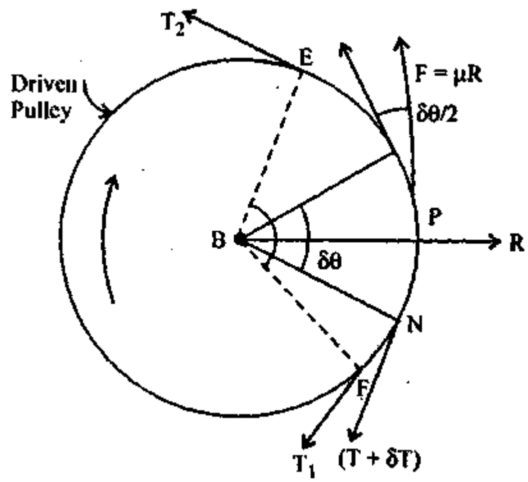
$$= (F_1 \sin\theta_1 + F_2 \sin\theta_2) \times x$$

$$= F_1 \times x \sin\theta_1 + F_2 \times x \sin\theta_2$$

$$R \times r = F_1 \times r_1 + F_2 \times r_2 \quad \textbf{Proved.}$$

(b) Derive an expression for the ratio of belt tensions in a flat belt drive.

Ans. Ratio of Belt Tension: For the equilibrium of elemental piece (MN). Resolving all the forces acting on the belt MN in horizontal direction.



$$R = T\sin\frac{\delta\theta}{2} + (T+\delta T)\sin\frac{\delta\theta}{2} \qquad \left\{\begin{array}{l}\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2} \\ \text{\& neglecting the small quantities}\end{array}\right.$$

So $\quad R = T \times \delta\theta \quad$ ...(1)

Resolving all the forces vertically

$$F = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \qquad \left(\cos\frac{\delta\theta}{2} = 1\right)$$

$$F = (T+\delta T) - T = \delta T$$

$$\mu R = \delta T \;\Rightarrow\; R = \frac{\delta T}{\mu} \quad ...(2)$$

F.om (1) and (2)

$$\frac{\delta T}{T} = \mu\delta\theta$$

$$\int_{T_1}^{T_2}\frac{\delta T}{T} = \int\mu.\delta\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu\theta}$$

(c) Explain briefly different types of friction.

Ans. Types of Friction:

1. Dry Friction: The friction between dry surfaces in contact is called dry friction. It is also called coulomb friction. It is subdivided into sliding friction and rolling friction.

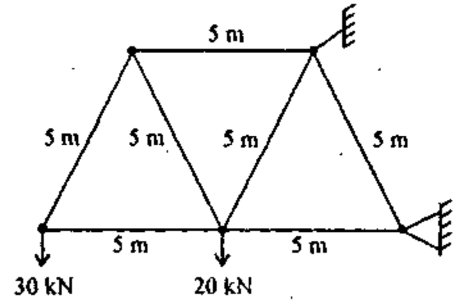
2. Fluid Friction: Fluid friction manifests when a lubricating fluids is introduced between the contact surfaces of two bodies.

3. Static and Dynamic Friction: The static friction is the frictional force that develops between making surfaces when subjected to external forces but there is no relative motion between them.

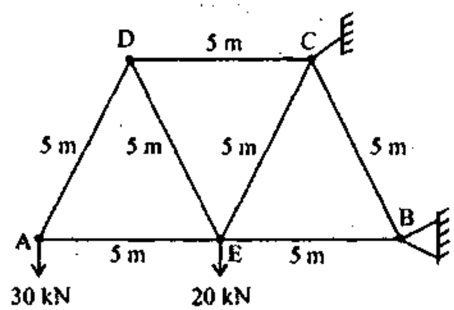
The dynamic friction is the frictional force that developes between making surfaces when subjected to external forces and there is relative motion between them. It is also known as kinetic friction.

Q.4. Answer any ONE part of the following: **(10)**

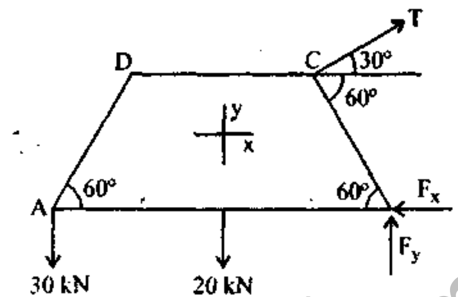
(a) Find the axial forces in all members of a truss as shown in figure.

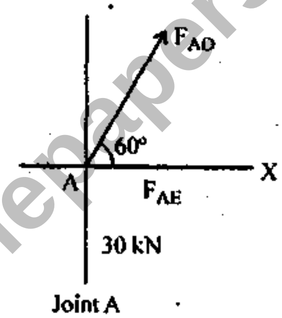


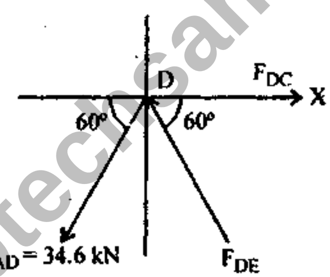
Ans.



Step 1. Free body diagram are drawn to represent the forces







Joint D

The reactions at the supports can be obtained from equation of equilibrium.

$\Sigma F_X = 0 \Rightarrow T\cos 30° - F_X = 0$ ...(1)

$\Sigma F_Y = 0 \Rightarrow T\sin 30° - F_Y - 20 - 30 = 0$ ...(2)

$\Sigma M_B = 0 \Rightarrow 5 \times T - 20 \times 5 - 30 \times 10 = 0$ ...(3)

From equation (3) $T = 80$ kN

From eq. (1), we get

$$F_X = 80\cos 30° = 69.282 \text{ kN}$$

From equation (2) $F_Y = 50 - 80\sin 30°$

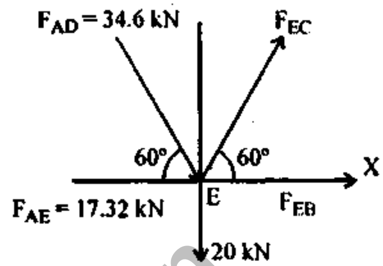
$$F_Y = 50 - 40 = 10 \text{ kN}$$

Calculation of unknown forces F_{AD} and F_{AE}

$\Sigma F_X = 0 \Rightarrow F_{AE} - F_{AD}\cos 60° = 0$ ...(4)

$\Sigma F_Y = 0 \Rightarrow F_{AD} \times \sin 60° - 30 = 0$ ...(5)

From eqn. (5)



$$F_{AD} = \frac{30}{\sin 60^\circ} = 34.6 \text{ kN/(J)} \quad \textbf{Ans.}$$

From eqn. (4) $\quad F_{AE} = 34.6 \times \cos 60^\circ$

$$F_{AE} = 17.32 \text{ kN} \quad \textbf{Ans.}$$

Considering the free body diagram of **Joint D.**

$$\Sigma F_X = 0 \quad F_{DE} \cos 60^\circ - F_{DC} + F_{AD} \cos 60^\circ = 0 \quad ...(6)$$

$$\Sigma F_Y = 0 \quad F_{AD} \sin 60^\circ - F_{DE} \sin 60^\circ = 0 \quad ...(7)$$

From (7) $\quad F_{DE} = \dfrac{F_{AD} \sin 60^\circ}{\sin 60^\circ} = 34.6 \text{ kN}$

$$F_{DE} = 34.6 \text{ kN}$$

$$34.6 \cos 60^\circ - F_{DC} + 34.6 \cos 60^\circ = 0$$

$$F_{DC} = 34.6 \text{ kN(T)}$$

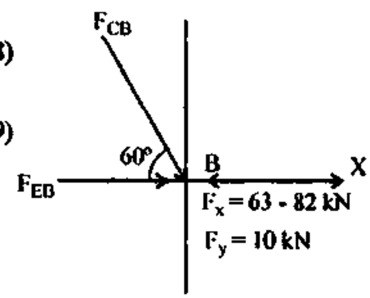
Joint 'E':

$\Sigma F_X = 0$

$$-F_{EB} + F_{AE} + F_{EC} \cos 60^\circ + F_{DE} \cos 60^\circ = 0 \quad ...(8)$$

$\Sigma F_Y = 0$

$$\Rightarrow \quad F_{DE} \sin 60^\circ + 20 - F_{EC} \sin 60^\circ = 0 \quad ...(9)$$



From eqn. (9) $\quad F_{EC} = \dfrac{20 + 34.6 \sin 60^\circ}{\sin 60^\circ} = \dfrac{45.56}{\sin 60^\circ}$

$$F_{EC} = 57.69 \text{ kN(T)}$$

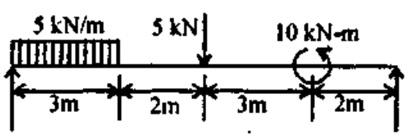
$$F_{EB} = 17.32 + 57.69 \cos 60^\circ + 34.6 \cos 60^\circ$$

$$F_{EB} = 17.32 + 28.84 + 17.3 = 6 = 46 \text{ kN(C)}$$

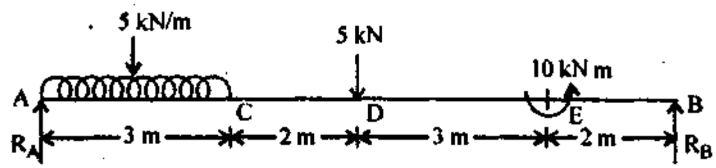
Joint 'B': $\quad F_{CB} = \dfrac{10}{\sin 60^\circ}$

$$F_{CB} = 11.54 \text{ kN (comp)}$$

(b) Draw the shear force and bending moment diagram for the beam loaded as shown in figure.



Ans.



$\Sigma F_Y = 0$

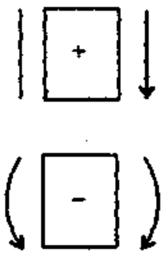
$$R_A + R_E = 20$$

$\Sigma M_A = 0$

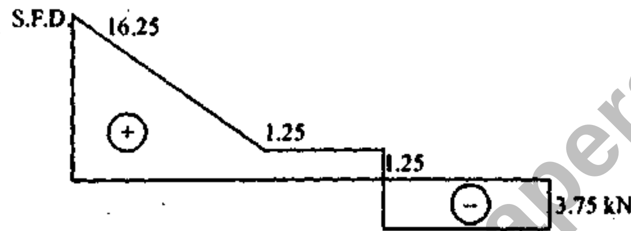
$$R_B \times 10 + 10 = 5 \times 5 + 5 \times 3 \times 3/2$$

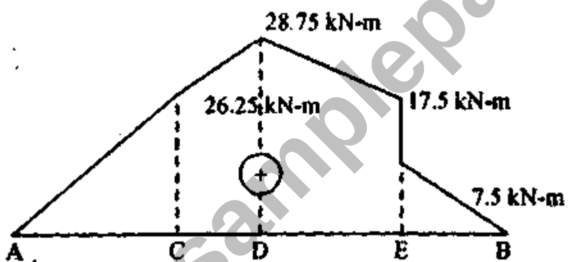
$$R_B = 3.75 \text{ kN}$$

$$R_A = 16.25 \text{ kN}$$



S.F.D.





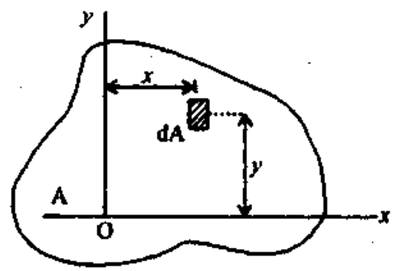
BMD

Q.5. Answer any TWO parts of the following: (2×5=10)

(a) Explain the following:

(i) Product of inertia

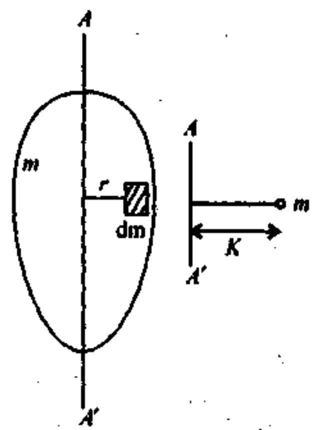
(ii) Mass moment of inertia



Ans. (i) Product of inertia: Consider a plane figure of area A in the $x = y$ plane as shown in figure. Divide this area into infinitesimal area. The integral

$$I_{xy} = \int xy\, dA$$

Obtained by multiplying each element dA of the area A by its coordinates x and y and integration extending over the entire area of the plane figure is called the product of Inertia (I_{xy}) of the figure with respect to the x and y area.

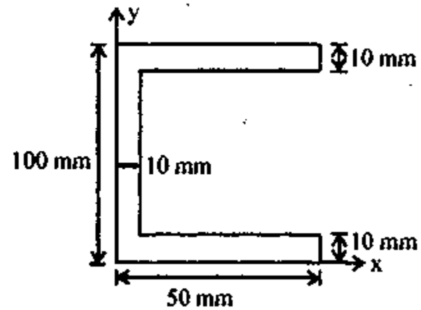


(ii) Mass moment of Inertia: Consider a body of mass m. The moment of inertia of the body with respect the axis AA' is defined by integral

$$I = \int r^2 dm$$

where, dm is the mass of an element of the body situated at a distance 'r' from the axis AA' and the integration is extended over the entire volume of the body.

(b) Locate the centroid of channel section as shown in Figure.



Ans.

$$A_1 = 50 \times 10 = 500 \text{ mm}^2$$
$$A_2 = 80 \times 10 = 800 \text{ mm}^2$$
$$A_3 = 50 \times 10 = 500 \text{ mm}^2$$
$$A = A_1 + A_2 + A_3 = 1800 \text{ mm}^2$$

$$\bar{x} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3}{A_1 + A_2 + A_3}$$

$$\bar{x} = 16.1 \text{ mm}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = 50 \text{ mm}$$

$\bar{x}_1 = 25$ mm

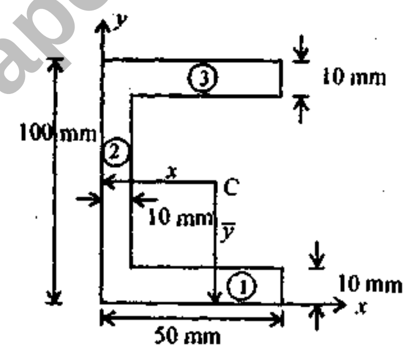
$\bar{y}_1 = 5$ mm

$\bar{x}_2 = 5$ mm

$\bar{y}_2 = 50$ mm

$\bar{x}_3 = 25$ mm

$\bar{y}_3 = 95$ mm



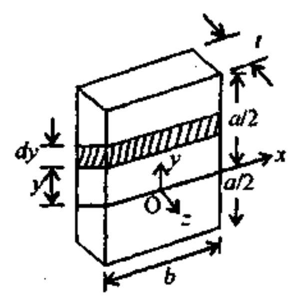
∴ Centroid of channel section $(\bar{x}, \bar{y})$ is (16.1, 50 mm).

(c) Determine the mass moment of inertia of a rectangular plate of size *axb* and thickness *t* about the centroidal axis.

Ans. Mass of the element

$$dm = \rho btdy$$
$$d_{I_{XX}} = y^2 dm$$
$$I_{XX} = \int dI_{XX} = \int_{-a/2}^{a/2} y^2 dm$$

$$= 2\int_0^{a/2} \rho bty^2 dy = 2\rho bt\left|\frac{y^3}{3}\right|_0^{a/2} = \frac{\rho bta^3}{12}$$



Mass of plate

$$M = \rho bat$$

$$I_{XX} = \frac{Ma^2}{12} \quad \text{Similarly} \quad I_{YY} = \frac{Mb^2}{12}$$

$$I_{ZZ} = I_{XX} + I_{YY} = \frac{Ma^2}{12} + \frac{Mb^2}{12} = \frac{M}{12}(a^2 + b^2)$$

Q.6. Answer any ONE part of the followng: (10)

(a) A train starts from rest and moves along a curved track of radius 600 m with uniform acceleration until it attains a velocity of 70 km/h at the end of third minute. Determine the tangential, normal and total acceleration of the train at the end of second minute.

Ans. $u = \omega_0 = 0$

$r = 600$ m

Speed after 3rd minutes = 70 km/h = $\frac{70 \times 5}{18}$ = 19.4 m/s

The train is moving with uniform acceleration hence using

$$\omega = \omega_0 + \alpha t$$

ω = angular velocity after 3 minute

$$\omega = \frac{19.4}{600} = 0.032 \text{ rad/s}$$

$$0.032 = 0 + \alpha \times 180 \quad (t = 180 \text{ sec})$$

$$\alpha = \frac{0.032}{180} = 0.000177 \text{ rad/sec}^2$$

Again using $\omega = \omega_0 + \alpha t = 0 + 0.000177 \times 120$

$\omega = 0.021$ rad/s

1. Now tangential acceleration

$$a_t = r \times \alpha = 600 \times 0.000177$$

$$a_t = 0.106 \text{ m/s}^2$$

2. Normal acceleration

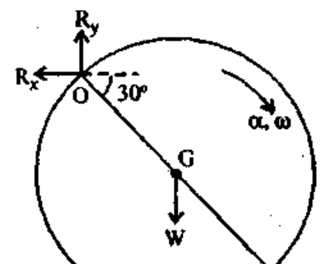
$$a_n = \omega^2 \times r = (0.021)^2 \times 6 \text{ m} = 0.264 \text{ m/s}^2$$

3. Total acceleration

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.106)^2 + 0.264^2}$$

or $a = 0.288 \text{ m/s}^2$

(b) The cylinder shown in figure is 70 cm in diameter and weighs 500 N. It is rotating about the fixed axis *O* and has an angular velocity of 7 rad/s at the given instant. Using D'Alembert's principle, find the horizontal and vertical components of the reaction at *O*.



Ans. $$I_G = \frac{1}{2}mr^2 = \frac{1}{2} \times \frac{500}{9.81}\left(\frac{0.70}{2}\right)^2 = 3.12 \text{ kg./m}^2$$

$$\Sigma M_O = 0$$

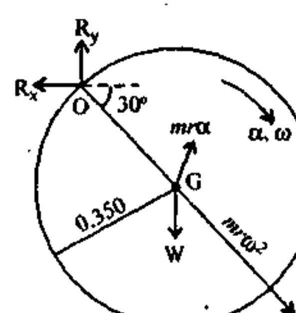
$$500 \cos 30° \times \frac{0.70}{2} - \left(\frac{500}{9.81} \times \frac{0.70}{2} \times \alpha\right) \times \frac{0.70}{2} - 3.12\alpha = 0$$

$$151.55 - 9.36\,\alpha = 0$$

$$\alpha = 16.19 \text{ rad/s}^2$$

$$m\bar{r}\alpha = \frac{500}{9.81} \times 0.35 \times 16.19 = 288.8 \text{ N}$$

$$m\bar{r}\omega^2 = \frac{500}{9.81} \times 0.35 \times (7)^2 = 874.10 \text{ N}$$



$\Sigma F_Y = 0$ gives

$$R_Y = 500 + 874.10 \sin 30° - 288.8 \cos 30°$$
$$R_Y = 500 + 437.05 - 250.10$$
$$R_Y = 686.95 \text{ N} \quad \textbf{Ans.}$$

$\Sigma F_X = 0 \qquad R_X = 874.10 \cos 30° + 288.8 \sin 30° = 756.99 + 144.4$

$$R_X = 901.39 \text{ N} \quad \textbf{Ans.}$$

Q.7. Answer any TWO part of the followng: (10)

(a) A 300 mm deep rectangalar beam is simply supported over a span of 6 m. What uniformly distributed load per meter the beam can carry if bending stress is not to exceed 110 N/mm². Take $I = 8.5 \times 10^6$ mm².

Ans. Given, depth = 300 mm, length = 6 m

Maximum bending stress $\sigma_{max} = 110 \text{ N/mm}^2$

$$I = 1.85 \times 10^6 \text{ mm}^2$$

$$\text{Maximum B.M.} = \frac{\omega \times L^2}{8} = \frac{\omega \times 6^2}{8} = \frac{36\omega}{8} = 4.5\,\omega \text{ Nm}$$

$$M = 4.5 \times 1000\,\omega \text{ Nmm} = 4500\,\omega \text{ Nmm}$$

Now $\quad M = \sigma_{max} \times Z \qquad$ Where $Z = \dfrac{I}{y_{max}} = \dfrac{8.5 \times 10^6}{150}$

$$M = 110 \times \frac{8.5 \times 10^6}{150}$$

$$4500\,\omega = \frac{110 \times 8.5 \times 10^6}{150}$$

$$\omega = \frac{110 \times 8.5 \times 10^6}{150 \times 4500} = 1385 \text{ N/m} \quad \textbf{Ans.}$$

(b) A rectangular bar of uniform cross-section 4 cm × 2.5 cm and of length 2.2 m is hanging vertically from a rigid support. It is subjected to axial tensile loading of 10 kN. If density of steel is 8000 kg/m³ and $E = 200$ GN/m², find the maximum stress and the elongation of the bar.

Ans. Given data C

Cross-sectional area $= 4 \times 2.5 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2 = 10 \times 10^2 \text{ mm}^2$

Length $L = 2.2$ m

Axial load = 10 kN

Density = 8000 kg/m³

$E = 200$ GN/m²

$(\delta l)_{Total}$ = (Elongation due to applied load) + (Elongation due to self wt of bar)

$$(\delta l)_{Total} = \frac{PL}{AE} + \frac{WL}{2AE}$$

Weight of bar = density × volume

$$W = 8000 \times 9.81 \times 10 \times 10^{-4} \times 2.2$$
$$W = 172.6 \text{ N}$$

$$\delta l = \frac{10 \times 10^3 \times 2200}{10 \times 10^2 \times 200 \times 10^3} + \frac{172.6 \times 2200}{2 \times 10 \times 10^2 \times 200 \times 10^3}$$

$= 0.11 + 0.000949 = 0.110$ mm **Ans.**

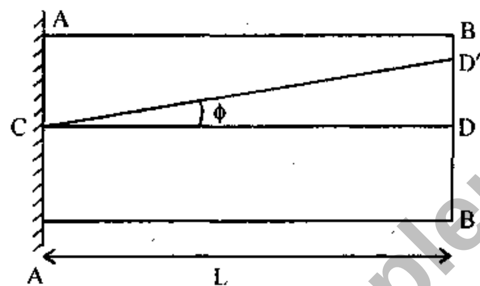
$$\boxed{\delta l = 0.110 \text{ mm}}$$

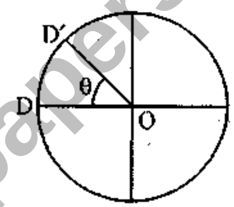
$$\text{Maximum stress} = \frac{\text{Total load}}{\text{Cross-sectional area}} = \frac{10 \times 10^3 + 172.6}{10 \times 10^2}$$

$$\sigma_{max} = 10.17 \text{ N/mm}^2$$

(c) Derive the torsion formula $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$.

Ans.





Shear strain at outer surface $= \frac{DD'}{L}$

$\therefore \quad \frac{DD'}{CD} = \phi$

And $\quad DD' = OD \times \theta = R \times \theta$

or $\quad \frac{R \times \theta}{L} = \phi$

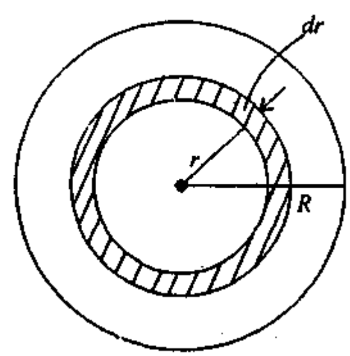
Modulus of rigidity $C = \frac{\tau}{(R\theta / L)}$

$$\frac{C\theta}{L} = \frac{\tau}{R}$$

$$\therefore \quad \tau = \frac{RC\theta}{L}$$

or $\quad \tau \propto R \quad$ or $\quad \frac{\tau}{R} =$ const.

if g is the shear stress induced at a radius 'r' from the centre of the shaft then



$$\frac{\tau}{R} = \frac{q}{r}$$

$$\frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r} \quad \ldots(A)$$

Shear stress of the radius r

$$q = \frac{\tau}{R} \times r$$

Turning force $= q \times dA = \frac{\tau}{R} \times r \times 2\pi r dr$

Turning moment

$$dT = \frac{\tau}{R} \times 2\pi r^3 dr$$

$$dT = \frac{\tau}{R} 2\pi r r^2 dr$$

$$dT = \frac{\tau}{R} r^2 dA$$

Total torque $\quad T = \int_0^R dT = \int_0^R \frac{\tau}{R} r^2 dA \quad$ or $\quad T = \frac{\tau}{R} \int_0^R r^2 dA$

where $r^2 dA$ = Moment of inertia of the elementary ring about an axis perpendicular to the plane of figure passing through the centre of circle.

$$\int_0^R r^2 A = \text{Polar moment of inertia}$$

$$J = \frac{\pi}{32} D^4$$

$$\therefore \quad T = \frac{\tau}{R} \times J \quad \text{or} \quad \frac{T}{J} = \frac{\tau}{R} \qquad \text{...(B)}$$

Combining equations (A) and (B)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \quad \text{proved.}$$