B.TECH. EME-102

FIRST ODD SEMESTER EXAMINATION, 2009-2010

ENGGINERING MECHANICS

Time : 3 Hours *Total Marks :* **100**

Note : (i) This paper is three sections. Section A carries 20 marks, Section B carries 30 marks and Section C carries 50 marks.

(ii) Attempt all questions. Marks are indicated against each question part.

(iii) Assume missing data suitably, if any.

1. Yon are required to answer ali the parts : (2×10=20)

Choose correct answer for the following parts :

(a) If number of forces act simultaneously on a particle, it is possible :

(i) not to replace them by a single force

(ii) to replace them by a single force

(iii) to replace them by a single coupie

(iv) to replace them by a force and couple

Ans. Both II and IV

(b) Moment of Intertia of a circular area, about an axis perpendicular to the area passing through its centre is given by :

(i) $\pi d^4/8$ (ii) $\pi d^4/16$

(iii) $\pi d^4/32$ (iv) $\pi d^4/64$

Ans. (iii) $\pi d^4/32$

Fill In the blanks for the foliowing three parts :

You will be awarded full marks, if all the matches in a part are correct otherwise will be awarded zero.

(c) In truss analysis, the weight of truss member is assumed to be .... and stress induced on application of force in truss members is ....

Ans. Zero, Axial

(d) Centripetal component of acceleration is measured .... to the direction of velocity and towards the centre of curvature of path.

Ans. Perpendicular, Normal

(e) The value of shear stress which is Induced In the shaft due to the applied torqne Is .... at the centre and .... at the circumference.

Ans. Zero, maximum

Match the columns for the following three parts :

You will be awarded full marks, if all the matches In a part are correct otherwise will he awarded zero.

(f) Match the following coiumns :

Column 1	Column II
(I) Statics	(P) Study of forces the causes motion
(ii) Dynamlcs	(Q) Study of forces in rigid bodies
(lil) Kinetics	(R) Study of displacement, velocity and acceieratlon

(iv) Kinematics (S) Study of forces in moving bodies

Ans. I – Q, II – S, III – P, IV – R

(g) Match the following columns :

Column I	Column II
(i) Lami's theorem	(P) Dynamic equilibrium of particle
(ii) Maxwell theorem	(Q) Principle of moments
(iii) D'Alembert's principle	(R) Equilibrium of three concurrent forces
(iv) Varignon's theorem	(S) Force analysis of truses

Ans. I – R, II – S, III – P, IV – Q.

(h) Match the following columns :

Column I	Column II
(i) Torsional rigidity	(P) EI
(ii) Section modulus	(Q) GJ
(iii) Torsional stiffness	(R) I/y
(iv) Flexural rigidity	(S) T/θ

Ans. I – Q, II – R, III – S, III – S, IV – P

Choose the correct answer for the following two parts :

(i) Two forces can be in equihrium only if they are :

(I) equal in magnitude

(II) opposite in direction

(III) collinear in action

(i) Only I and II are correct

(ii) Only I and III are correct

(iii) Only II and III are correct

(iv) All are correct

Ans. (iv) All are correct

(j) For the same power transmitted :

(I) the weight of solid shaft is less than that of the hollow shaft

(II) the weight of hollow shaft is less than that of the solid shaft

(III) No relation exists between power transmitted and the weight of solid and hollow shaft

(i) Only I and III are correct

(ii) Only II and III are correct

(iii) II alone is correct

(iv) I alone is correct

Ans. (iii) II alone is correct

2. Answer any three parts of the following :

(a) Three cylinders A, B and C each weighing 100 N and diameter 80 mm are placed in a channel of 180 mm width as shown in Fig. I. Determine the pressure exerted by the cylinder A and B at the point of contact.

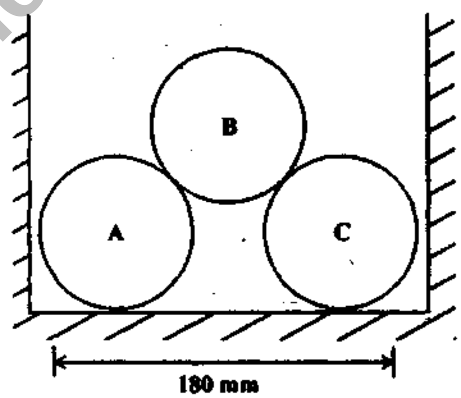
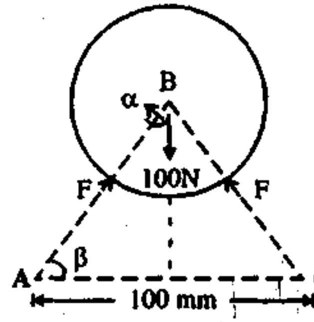
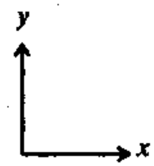


Fig. I

Ans. Free body diagram is should blo



Due to symmetry, reactions of A and C on B are same,



$$\cos\beta = \frac{50}{80} \Rightarrow \beta = \cos^{-1}(5/8) = 51.33°$$

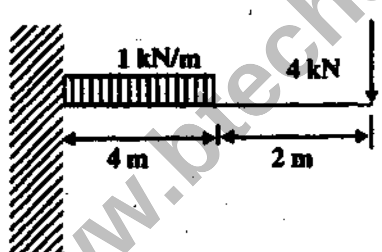
$$\alpha = (90 - 51.33) = 38.7°$$

For the equilibrium of B

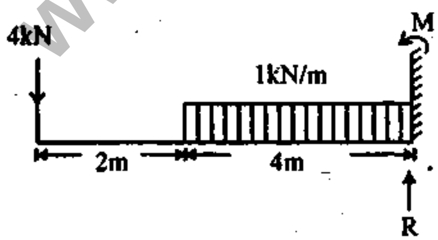
$$2F\cos\alpha = 100 \text{ N}.$$

$$\text{or } F = \frac{100}{2\cos(38.7)} = 64 \text{ N Ans.}$$

(b) Calculate the values of shear force and bending moments for the cantilever beam shown in Fig. 2. Also draw the shear force and bending moment diagrams.

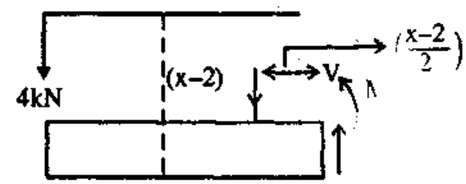


Ans.



R = 4 + 4 × 1 = 8 kN.

M = (– 1 × 4 × 2 – 4 × 6) = – 32 KNM.



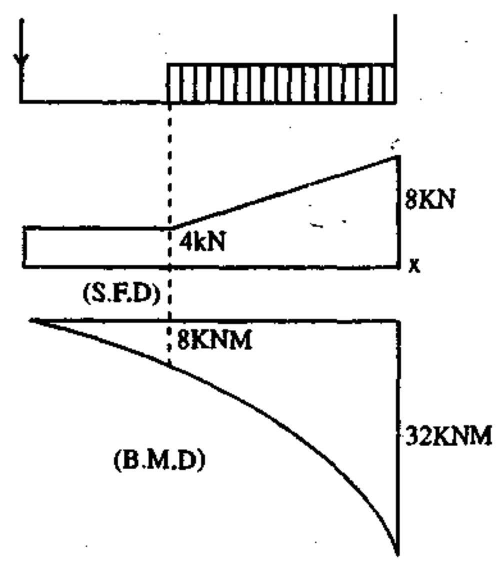
$\Sigma F_4 = 0$ Given.

Shear Force $\quad V = 4 + \langle x - 2 \rangle$

and $M = -4x - \dfrac{\langle x-2 \rangle}{2}$

$$V = \begin{cases} 4 & x < 2 \\ 2 + x & x > 2 \end{cases}$$

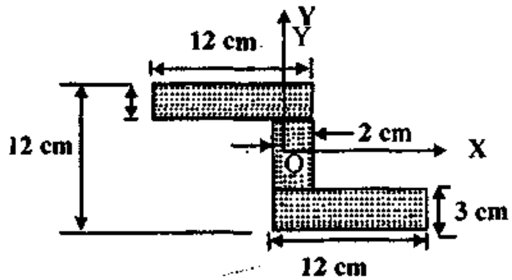
$$M = \begin{cases} -4x & x < 2 \\ -4x - \left(\dfrac{x-2}{2}\right)^2 & x > 2 \end{cases}$$



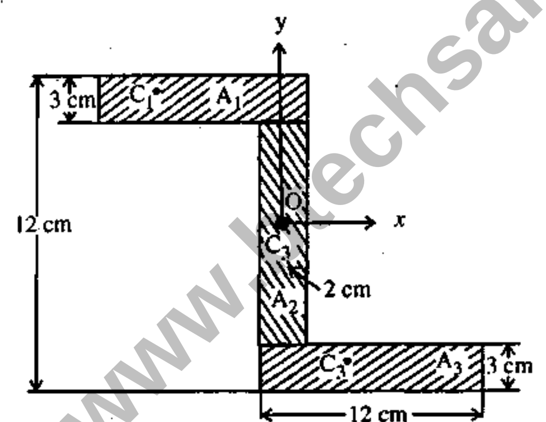
(c) For the z-section as shown in Fig. 3, the moment of inertia with respect to *x* and *y* axes are given as

I_x = 1548 cm⁴ and I_y = 2668 cm⁴

Determine the principal axes of the section about O (centroid of vertical section and point O coincides) and values of the principal moments of inertia.



Ans. Area of Z-section can be considered to be made up of three rectangles A_1, A_2 and A_3 with their centroids at C_1, C_2 and C_3 respectively. Note that C_2 and O are coincident points.



Area (cm²)	Distance of the centroid from x and y axes (cm)	
A_1 : 12 × 3 = 36	$\bar{x}_1$ = – 5.0	y1 = + 4.5
A_2 : 2 × 6 = 12	$\bar{x}_2$ = 0	y_2 = 0
A_3 : 12 × 3 = 36	$\bar{x}_3$ = + 5	y_t = – 45

Product of inertia of the total area.

I_n = [0 + 36 (–50) (4.5)] + [0 + 0] + [0 + 36(5.0) (–4.5)]

I_n = – 1620 cm⁴

(Using parallel axis theorem and the concept that product of inertia vanishes if any one of the axes is the axis of symmetry).

I_x = 1548 cm⁴. I_y = 2668 cm⁴ (given)

For finding the directions of the principle axes.

$$\tan 2\theta_m = \frac{2I_{xy}}{I_y - I_x} = \frac{-2\times1620}{2668-1548}$$

$\tan 2\theta_m$ – 2.893

$2\theta_m$ = – 70.93°, θ_m = 35.46°

θ_m = – 35.46°, and + 54.54°

$$I_{max,min} = \frac{I_1 + I_2}{2} \pm \sqrt{\left(\frac{I_1 - I_2}{2}\right)^2 + (I_{xy})^2}$$

$$= \frac{1548 + 2668}{2} \pm \sqrt{\left(\frac{1548 - 2668}{2}\right)^2 + (1620)^2}$$

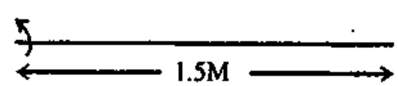
= 2108 ± 1714

I_{max} = 3822 cm⁴

I_{min} = 394 cm⁴

(d) A borizontal bar 1.5 m long and of small cross-section rotates about vertical axis through one end. It accelerates uniformly from 1200 rpm to 1500 rpm In an interval of 5 seconds. What is the linear velocity at the beginning and end of the Interval? What are the normal and tangential components of acceleration of the mid point of the bar after 5 seconds after the acceleration begins?

Ans.



Horizontal Rod

Given ω_0 = 1200 RPM = 125.7 Rad/sec, at t = 0

ω_2 = 1500 RPM = 157.1 Red/sec at t = 5 sec.

We have to find V(0) and V(5).

$$\omega_2 = \omega_0 + \alpha t \Rightarrow \alpha = \left(\frac{\omega_2 - \omega_0}{t}\right)$$

or $$\alpha = \left(\frac{157.1 - 125.7}{5}\right) = 6.28 \text{ Rad/sec}^2$$

$\therefore$ V_0 (= velocity at beginning) = 125.7 R m/sc
= 125.7 × 0.75
= 94.27 m/s

$V_5 = 157.1 \times 0.75 = 117.825$

at $t = 5$, Transiential accelerations

$$a_t = R\alpha = \frac{1.5}{2} \times 6.28 = 4.71 \text{ m/s}^2$$

Normal acceleration

$a_n = -\omega^2 R = -(157.1)^2 \times 0.75$

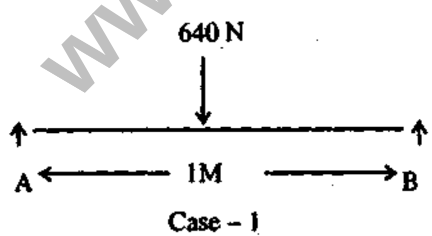
or $a_n = -1.85 \times 10^4$ m/sec². Ans

(e) A cast iron test beam 20 mm × 20 mm in section and 1 m long and supported at the ends fails when a central load of 640 N is applied. What uniformly distributed load will break a cantilever of the same material 50 mm wide, 100 mm deep and 2 m long?

Ans. Case-I : Length of S.S beam = 1m

Cross section = 20mm × 20mm

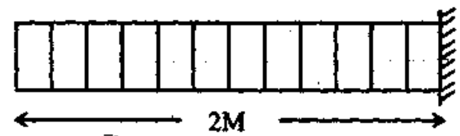
Central Load = 640 N.



Case – 1

Case-II : Length of U.D.L beam = 2m. Cross section of U.D.L beam = 50 × 100 mm.

Material of beam for both the cases is same.



Case-I

$$M_{max} = \frac{640}{2} \times 500 \, Nmm = 1.6 \times 10^5 \, N.mm$$

$$Z = \frac{20 \times (20)^2}{6} = \frac{4}{3} \times 10^3 \, mm^3 = 1.33 \times 10^3 \, mm^3$$

$$\sigma_{max}\left(\frac{1.6 \times 10^5 \times 3}{4 \times 10^3}\right) = 120 \text{ N/}mm^2.$$

Hence strength of material = 120 N/mm^2.

Case II

$$Z = \frac{bb^2}{6} = \frac{50 \times 100^2}{6} = 8.33 \times 10^4 \, mm^3$$

$$\frac{M_{max}}{Z} = 120 \, N/mm^2$$

$$M_{max} = 120 \times 8.333 \times 10^4$$

$$= 9.99 \times 10^6 \text{ N.}mm$$

$$\frac{\omega l^2}{2} = 9.99 \times 10^6$$

ω = 5 N/mm = 5 kN/m.

where ω = uniform distributed loading intensity.

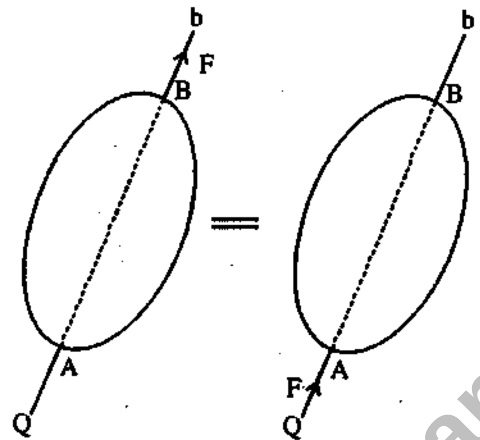
SECTION-C

3. Answer any two parts of the following : (5×2=10)

(a) Explain the theorem of transmissibility of a force. What are its limitations?

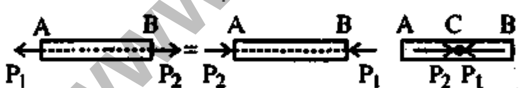
Ans. Principle of Transmissibillity of a Force : It states that the condition of equilibrium or of motion of rigid body will remain unchanged if the point of application of a force acting on the rigid body is transmitted to act at any other point along its line of action.

A force F acting on the rigid body at point B, having the line of action ab, can be replaced by the same force F but acting at the point A provided this new point A lies along the line of action ab of the force.



In other words, the force F acting at point A can be transmitted to act any other point B along its line of action without changing its effect on the rigid body.

Next consider a prismatic bar AB which is acted upon by two equal and opposite coaxial forces P_1 and P_2 as shown in Fig. (a)



(a) Pull (b) Push © No force

Using the principle of transmissibility, the force P1 acting at point A can be transmitted to act at point B can be transmitted to act at A. This new system of forces [Fig. (b)] acting on the bar, as obtained by the principle of transmissibility, does not change the condition of equilibrium of the body.

∴ Limitations :

1. This theorem can be applied for rigid bodies only.
2. It can not be used to determine the Internal forces ad deformations of the body.

(b) Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 rpm. The coefficient of friction between belt and pulley is 0.25, angle of lap 160° and maximum tension in the belt is 2.5 kN.

Ans. Diameter of pulley D = 600 mm.

Rotation N = 200 RPM

Hence $\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 200}{60} = 2.9$ Rod/see

$T_1 = 2.5$ KN.

$\theta = \frac{160\pi}{180} = 2.797$ Rod and

$\mu = 0.256$

From Relation

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{(0.25 \times 2.793)}$$

or $$\frac{T_1}{T_2} = 2.01$$

$$T_2 = \frac{2.5}{2.01} = 1.244 \text{ KN.}$$

Torque = (T1 – T2)R

$$= (2.5 - 1.244) \times 10^3_N \times \frac{300}{1000} m$$

Torque = 376.8 NM

Power P = Tω = 376.8 × 20.94 = 7.89 × 10^3ω

or P = 7.89 KW.

(c) Fig. 4 shows a system of levers supporting a load of 500 N. Determine the reactions at the supports A and B.

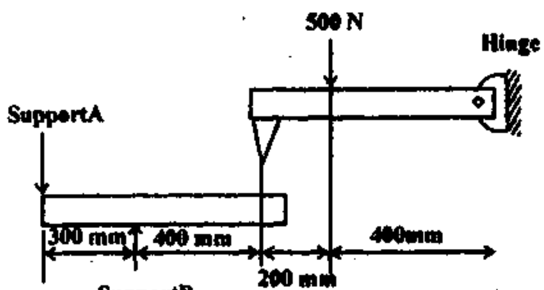
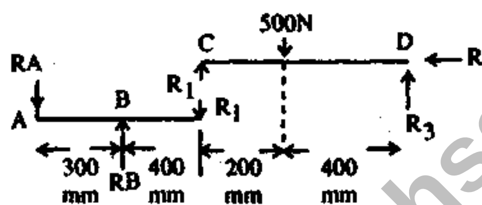


Fig. 4

Ans. Free body diagram of the system is shown below



$\Sigma M_D = 0$ Gives

$R_1 \times 600 = 500 \times 400 \Rightarrow R1 = 333.33$ N. ...(1)

$\Sigma M_D = 0$ Gives

$RA \times 300 = R1 \times 400 \Rightarrow R_A = \frac{400 R_1}{300}$ ...(2)

From (1) and (2)

$$R_A \frac{400 \times 333.3}{300} = 444.4 \text{ N}$$

$\Sigma F_4 = 0$ Gives

$R_B = R_A + R_1 = 444.4 + 333.33$

or $R_B = 777.7$ N

$\therefore \left. \begin{array}{l} R_A = 444.4 \text{ N} \\ R_B = 777.7 \text{ N} \end{array} \right\}$

4. Answer any one part of the following :

(a) Find the forces in all members of a truss as shown in Fig. 5 which carries a horizontal load of 12 kN at point D and vertical load of 18 kN at point C.

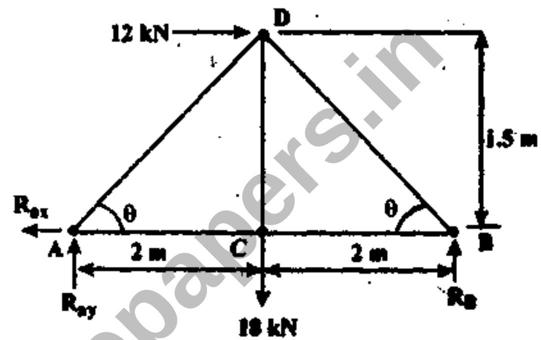
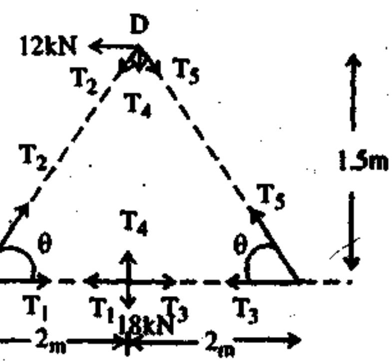


Fig. 6

Ans. Different tensile/compressive forces are shown in following F.B.D.



$\tan(\theta) = \tan(A) = \frac{1.5}{2} \Rightarrow \theta A = \tan^{-1}(0.75)$

$= 36.07$

$\beta = 90 - 36.87 = 53.13$

$\Sigma M_A = 0$ Gives $12 \times 1.5 + 18 \times 2$

or $R_B = 13.5$ KN.

$\Sigma f_x = 0$ Gives $R_{AX} = 12$ KN.

and $\Sigma F_4 = 0$ Gives $R_{AY} = 4.5$ KN.

Taking Joint 'A', S fy = 0 Gives

$T_2 \sin g\ \theta + R_{AY} = 0$

$$T_2 = -\frac{4.5}{\sin 36.87} = -7.5\text{KN}$$

S f_x = 0 Gives $R_{AX} = T_1 + T_2 \cos\theta$

or $T_1 = 12 + 7.5 \cos 36.87 = 18$ KN.

Taking Joint C

$\Sigma f_x = 0$ Gives, $T_3 = T_1 = 18$ KN.

Taking Joint B

$\Sigma f_y = 0$, Gives

$TS \sin\theta + T_3 = 0$

$$\text{or } TS = -\frac{13.5}{\sin 36.87} = -22.5 \text{ KN}$$

Taking Joint B

$\Sigma f_y = 0$, Gives

$TS \sin\theta + T_3 = 0$

$$\text{or } TS = -\frac{13.5}{\sin 36.87} = -22.5 \text{ KN}$$

Force in Different members are shown below :

AC = 18 KN

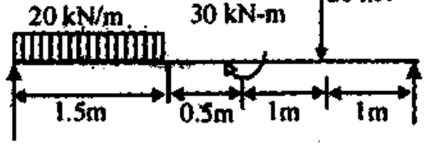
CB = 18 KN

CD = 18 N

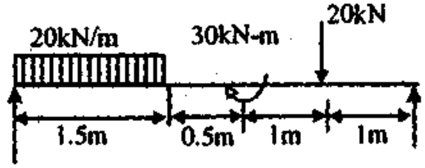
AD = – 7.5 KN

BD = – 22.5 KN

(b) A beam is loaded as shown in Fig. 6. Draw its shear force and bending moment diagram.



Ans.



Taking moment about 'A' we get

$20 \times 1.5 \times 0.75 + 30 + 20 \times 3 - 4\ R_B = 0$

RB = 28.125 kN.

Also $R_A + R_B = 20 \times 1.5 + 20$

$\Rightarrow \quad R_A = 50 - 28.125 = 21.87$ kN.

For Region $0 < x < 1.5$

Shear force $V = 21.876 - 20\,x$

at $x = 0$, $V = 21.87 - 0 = 21.87$ kN

$$V = 0, \text{ at } x = \frac{21.87}{20} = 1.09\,m$$

at $x = 1.5$, $V_{1.5} = 21.87 - 20 \times 10.5$

or $V_{1.5} = -8.13$ kN.

Bending Moment $M = 21.87\,x - 10x^2$

at $x = 0$, $m = 0$

at $x = 1.09$, M = 11.96 kN.M

at $x = 1.5$, M = 10.31 kN.m

For the region $1.5 < x < 2$

Shear force V = – 8.13 kN.

Bending moment

$M = 28.13\,(4 - x) - 30 - 20(3 - x)$

$M = 22.52 - 8.13x$

$M\,(x = 1.5) = 22.52 - 8.13 \times 1.5 = 10.33$ kN.m

$M\,(x = 2) = 22.52 - 8.13 \times 2 = 6.26$ kN.m

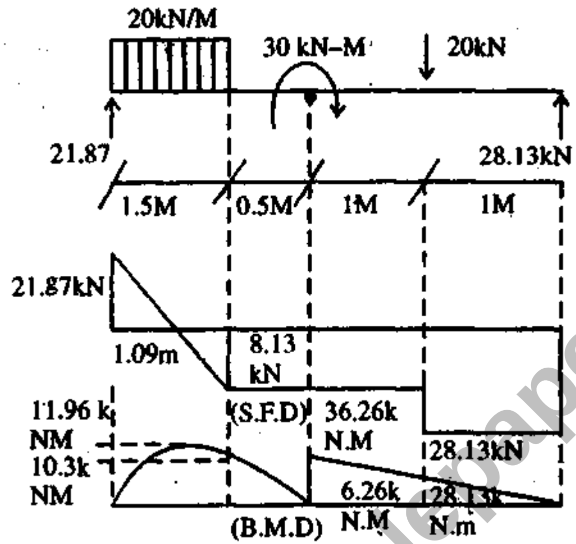
For the region $3 < x < 4$

Shear force V = – 8.13 – 20 = 28.13 kN.

Bending Moment $M = 28.13\,(4 - x)$

$M\,(\text{at } x = 3^+) = 28.13$

$M\,(\text{at } x = 4) = 0$



5. **Answer any two parts of the following :** **(5 × 2 =10)**

(a) **Explain the following :**

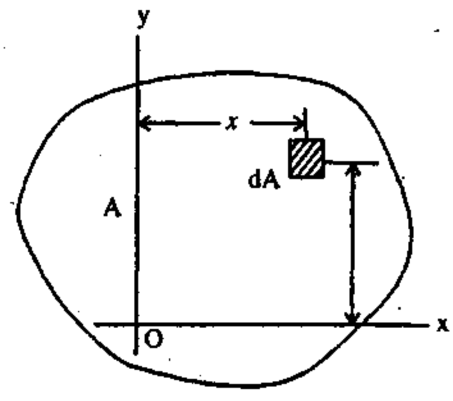
(i) **Product of inertia**

(ii) **Principal moment of inertia.**

Ans. (i) Product of Inertia : Consider a plane figure of area A in the x-y plane as shown in Fig. Divide this area into infinitesimal areas. The integral

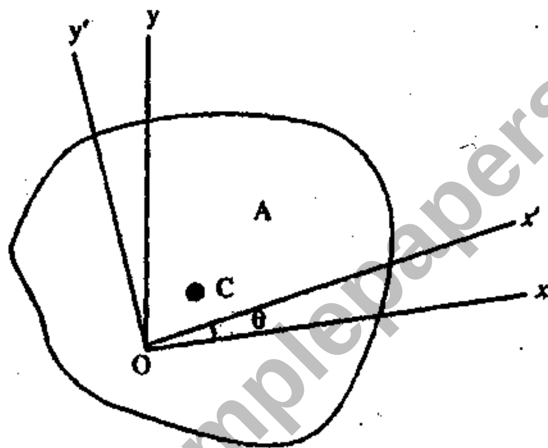
$$I_{xy} = \int xy\, dA \quad \text{....(1)}$$

obtained by multiplying each element dA of the area A by its coordinates x and y and the integration extending over the entire area of the plane figure is called the product of



Inertia (I_{x4}) of the figure with respect to the x and 'y' ares.

(ii) Principal moment of Inertia : It can be shown that the product of inertia I_{xy} during rotation of the axes changes its sign and becomes negative. From the above fact it can be concluded that there must be certain directios of the axes for which the product of inertia is zero. The axes taken in these directions are called the principal axes of the area. The two principal axes are perpendicular to each other and are such that the product of inertia of the given area with respect to these axes is zero.



Consider a plane figure of area A. Let the moments of inertia I_x, I_y and the product of inertia I_{xy} with respect to the axes x and y passing through any point O within or outside the area be known.

Let the axes be rotated anticlokwise about O by angle θ to new position x' and y'. It can be shown that the moments of inertia $I_{x'}$ and $I_{y'}$ and the product of inertia $I_{x'}y'$ about the axes x' and y' are given by the equations.

$$I'_x = \frac{I_x + I_2}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta \qquad ...(1)$$

$$I'_y = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta \qquad ...(2)$$

$$I'_x I'_y = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta \qquad ...(3)$$

Axes x' and y' corresponding to a zero value of product of inertia can be obtained by setting equation to zero. Let that angle be denoted by θ.

$$0 = \frac{I_x - I_y}{2}\sin 2\theta_m + I_{xy}\cos 2\theta_m \qquad ...(4)$$

Or tan $\quad 2\theta_m = -\dfrac{2I_{xy}}{I_x - I_y} = \dfrac{2I_{xy}}{I_y - I_x}$

Above expression can be used to determine the directions of the principle axes through O. This equation defies two values of $2\theta_m$ which are 180° apart. Thus the values of θ_m are 90° apart. The moments of inertia about these axes are called principal moments of inertia and are given by:

$$I_{min} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + (I_{xy})^2} \quad ...(5)$$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + (I_{xy})^2}$$

(b) A semicircular area is removed from the trapezoid as shown in Fig. 7. Determine the centroid of remaining area :

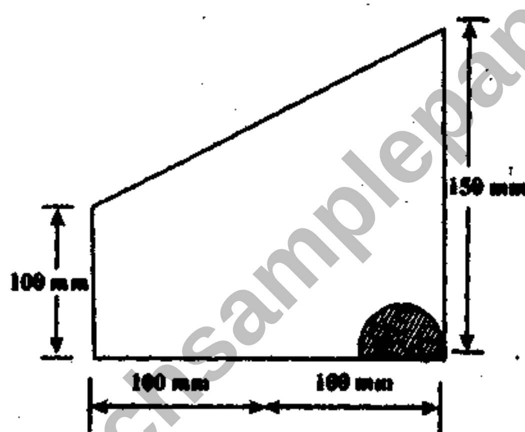
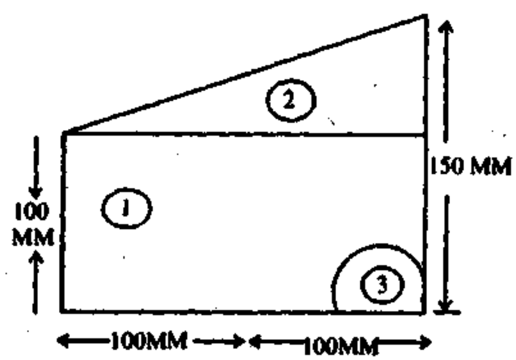


Fig. 7

Ans. Area of (1), $A_1 = 100 \times 200 = 2 \times 10^4$ mm

Centroids of (1) = (100,50)



Similarly

$A_2 = \frac{1}{2} \times 200 \times 50 = 5 \times 10^3 mm^2$, and (133.3, 116.7)

$A_3 = \frac{\pi \times 50^2}{2} = 3.927 \times 10^3\ mm^2$ and (150, 21.22)

$x_1 = \frac{200}{2} = 100$ $\quad$ $y_1 = \frac{100}{2} = 56$

$x_2 = \frac{2 \times 200}{3} = 133.33$ $\quad$ $y_2 = 100 + \frac{50}{3} = 116.7$

$x_3 = 150$ $\quad$ $y_3 = \frac{4 \times 50}{3\pi} = 21.22$

$$x_c = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 - A_3}$$

$$x_c = \left(\frac{2 \times 10^4 \times 100 + 5 \times 10^3 \times 133.3 - 3.93 \times 10^3 \times 150}{2 \times 10^4 + 5 \times 10^3 - 3.93 \times 10^3} \right)$$

$x_c = 98.59\ mm$

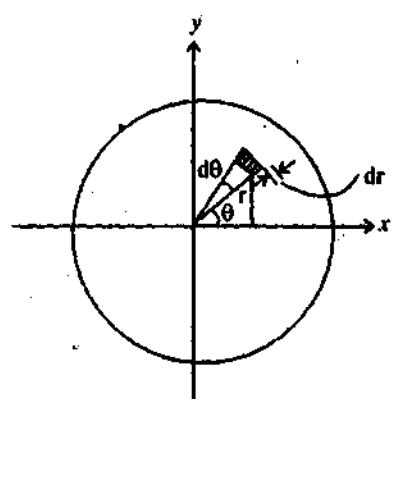
Similarly $y_c = \left(\frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - y_3} \right) = 71.2\ mm$

$x_c = 98.59$ mm

$y_c = 71.2$ mm

(c) Derive as expression of mass moment of inertia of a circular lamina about the central axis.

Ans. Consider an elemental area $r\ d\theta$ dr and thickness t ass shown in Fig.



dm = mass of the element = $\rho r \, d\theta \, dr \, t = \rho \, t \, r \, d\theta \, dr$

Its distance from x axis = $r \sin \theta$

$$\therefore \quad I_{xx} = \oint (r \sin \theta)^2 \, dm$$

$$= \int_0^R \int_0^{2\pi} r^2 \sin^2 \theta \rho \, t \, r \, d\theta \, dr$$

$$= \rho t \int_0^R \int_0^{2\pi} r^3 \left(\frac{1 - \cos 2\theta}{2} \right) dr d\theta$$

$$= \rho t \int_0^R \frac{r^3}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi} dr$$

$$= \rho t \int_0^R \frac{r^3}{2} \times 2\pi \, dr$$

$$= \rho t \pi \left[\frac{r^4}{4} \right]_0^R$$

$$= \rho t \frac{\pi R^4}{4}$$

Mass of the plate $M = \rho \times \pi R^2 t$

$$\therefore \quad I_{xx} = \frac{MR^2}{4}$$

Similarly, $I_{yy} = \dfrac{MR^2}{4}$

Actually $I = \dfrac{MR^2}{4}$ is moment of inertia of circular plate about any diametral axis in the plate To fin I_{zz} consider the same element.

$$I_{zz} \oint r_R^2 \, dm = \int_0^R \int_0^{2\pi} r^2 \rho \, t \, r \, dr \, d\theta$$

$$= \rho t \int_0^R r^3 \left[\theta \right]_0^{2\pi} dr$$

$$= \rho t \int_0^R 2\pi r^3 \, dr$$

$$= \rho t \; 2\pi \left[\frac{r^4}{4} \right]_0^R$$

$$= \rho t 2\pi \frac{R^4}{4} = \rho t \frac{\pi R^4}{2}$$

But total mass $M = \rho t p R^2$

$$I_{zz} = \frac{MR^2}{2}$$

6. Answer any one of the following :

(a) A cord is wrapped around a wheel of radius 0.2 m, which is initially at rest as shown in Fig. 8. If a force is applied to the cord ad gives it an acceleration a = (4t) m/sec^2, where t is in second. Determine the angular velocity of the wheel and the angular position of line OP both as a function of time.

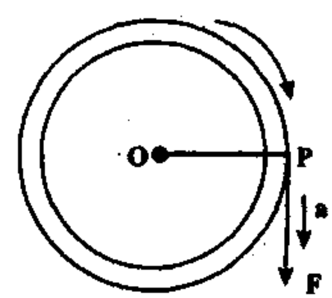


Fig.

Ans. Radios of wheel R = 0.2 m

Initial velocity V = 0 and a ~ 0.

After applying force acceleration a = 4(4t) m/sec^2.

$$\frac{dv}{dt}=4t \qquad \text{or } V=\frac{4t^2}{2}=2t^2+C_1$$

at $t = 0$, $v = 0$ gives $c_1 = 0$

$\therefore$ $V = 2t^2$

$$\frac{dx}{dt}=V=2t^2 \Rightarrow x=\frac{2t^3}{3}+C_2$$

At $t = 0$, $x = 0$, this Gives $C_2 = 0$

$$\therefore \quad x=\frac{2t^3}{3}$$

at $t = 5$, $V = 2 \times 5^2 = 50$ m/sec

Also, V = Rω = 50 m/sec

$$\Rightarrow \omega=\frac{50}{R}=\frac{50}{0.2}=250 \text{ Rod sec}$$

$$\omega R = V = 2t^2 \Rightarrow 0.2\omega = 2t^2 \Rightarrow \omega = 10t^2$$

$$R\theta = x = \frac{2}{3}+3 \Rightarrow 0.2\theta=\frac{2t^3}{3} \Rightarrow \theta=\frac{10t^3}{3}$$

(b) A road roller has a total mass of 12000 kg. The front roller has a mass of 2000 kg, a radius of gyration of 0.4 m and a diameter of 1.2 m. The rear axle, together with its wheels, has a mass of 2500 kg, a radius of gyration of 0.6 m and a diameter of 1.5 m. Calculate kinetic energy of rotation of the wheels and axles at a speed of 9 km/h and total kinetic energy of road roller.

Ans. Total mass M = 12000 kg.

Mass of front whal m_f = 2000 kg.

Radios Gyration of front whell k_f = 0.4m

Radios of Front wheel R_f = 0.6m.

Mass of Real Wheel M_r = 2500 kg.

Radios Gyration of Rear wheel kr = 0.6 m

Radios of Rear Wheel R_r = 0.75 m

$$V_c=\frac{9\,km}{kr}=\frac{9000\,m}{3600\,\text{sec}}=2.5\,m/\text{sec}$$

$$\omega_f=\frac{V_C}{R_f}=\frac{2.5}{0.6}=4.17 \text{ Rod/sec}$$

$$\omega_r=\frac{V_C}{R_r}=\frac{2.5}{0.75}=3.33 \text{ Rod/sec}$$

Kinetic energy (K.E) =

$$\frac{1}{2}mV_c^2+\frac{1}{2}mf\,k_f\,\omega_f^2+\frac{1}{2}M_f\,k_f\,\omega_f^2+\frac{1}{2}M_r\,k_r^2\,\omega_r^2$$

$$=\frac{1}{2}(12000)\times(2.5)^2+\frac{1}{2}\times 2000\times(0.4)^2\times(4.17)^2$$

$$+\frac{1}{2}\times 2500\times(0.6)^2\times(3.33)^2$$

$$=3.75\times10^4+0.2782\times10^4+0.499\times10^4$$

$$= 4.527 \times 10^4 \text{ J}$$

7. Answer any one of the following :

(a) Write the assumptions made in the theory of simple bending.

A beam of I-section is 250 mm deep. The flanges ore 15 mm thick, 100 mm wide while the web is 8 mm thick. Compare the flexural strength of this beam section with a rectangular section of the same material and area whose width is two-third depth.

Ans. 1. Beam is Initially straight

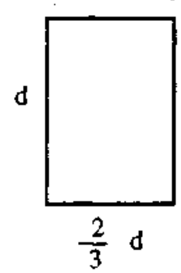
2. The beam has constant cross sectional area with an axis of symmetry.

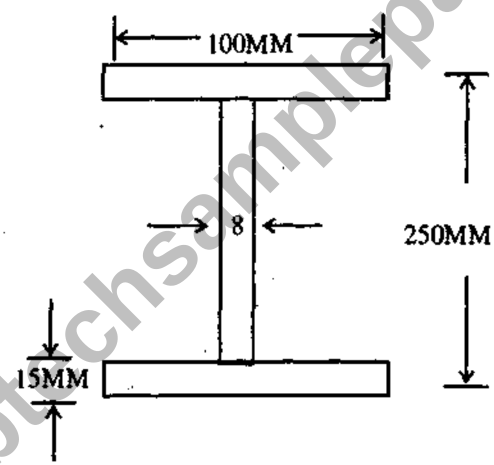
3. Material of Beam is Homogeneous. i.e., of the same density throughout and isotropic i.e., equally elastic in all directions.

4. Hooke's law is obeyed at all point i.e., stress is proportional to strain

5. Plane transverse section remains plane and normal after bending i.e., there is no distortion of the cross-section.
6. Every layer of material is free to expand or contract longitudinally and laterally under stress and do not exert pressure upon each other.
7. The value of youn's modulus (E) for the material is same in Tension and in compression.

Numerical part :





moment of Inertia of I beam

$$=\left[\frac{100\times(250)^3}{12}-\frac{92\times(220)^3}{12}\right]=4.857+10\ mm$$

Area of I section = $250 \times 100 - 92 \times 220$

$= 4.76 \times 10^3\ mm^2$

Area of Rectangle = $\frac{2}{3}d\times d=4.76\times10^3$ (Gives)

$\therefore\ d = 84.5$ mm

width of rectangular section $b = \frac{2}{3} d = 56.33$ mm

Moment of Inertia of Rectangle

$$= \frac{56.33 \times (84.5)^3}{11}$$

$$= 2.83 \times 10^6 \text{ mm}^4.$$

$$Z_1 \text{ (I–section)} = \frac{4.857 \times 10^7}{125} = 3.886 \times 10^5 \text{ mm}^3$$

$$Z_2 \text{ (rectangle)} = \frac{2.83 \times 10^6}{42.25} = 6.698 \times 10^4 \, mm^4$$

Ratio of flexural strength $\frac{EI_1}{EI_2} = \frac{\sigma_1 z_1}{\alpha_1 z_2} = \frac{Z_1}{Z_2}$

$$\frac{Z_1}{Z_2} = \left(\frac{3.886 \times 10^5}{6.698 \times 10^4}\right) = 5.8$$

(b) Prove that shear stress due to pure torsion is directly proportional to the radius of the shaft. The average torque transmitted by a shaft is 2255 Nm. The maximum torque is 146% of average torque. If the allowable shear stress in the shaft material is 45 N/mm², determine the suitable diameter of the shaft.

Ans. Given

$T_{av} = 2255$ Nm.

$$T_{max} = \left(\frac{146}{100}\right) \times T_{av} = 1.46 \times 2256 \text{ Nm}$$

$T_{max} = 3293.76 \text{ Nm} = 3.29 \times 10^6$ Nmm

Z_{max} (allowable shear stress) = 45 N/mm²

at d is diameter of shaft.

Then $\frac{16\,T}{\pi\,d^3} \le 45$

or $d^3 \ge \frac{16\,T}{45\,\pi} = \left(\frac{16 \times 3.29 \times 10^6}{45\pi}\right)$

or $d^3 > 3.72 \times 10^5 \text{ mm}^3$

or $d > 71.96$ mm

or $d \ge 72$ mm