B.Tech.

EME-202

SECOND SEMESTER THEORY EXAMINATION, 2010-11

ENGINEERING MECHANICS

Time: 3 *Hours* *Total Marks:* 100

Note: *Attempt all questions.*

Note: (i) *This paper is in three Sections. A carriers 20 marks, Section-B carrie 30 marks and Section-C carries 50 marks.*

(ii) *Attempt all questions. Marks are indicated against each question part.*

(iii) *Assume missing data suitably, if any.*

SECTION-A

You are required to answer all the parts. **(2×10=20)**

1. Choose correct answer for the following parts:

Q. 1(*a*) If two forces *P* and *Q* acting at a point are represented in magnitude and direction by the two adjacent sides of the parallelogram and θ is the angle between the forces then:

(*i*) if $\theta = 90°$, $R = 2P\cos(\theta/2)$

(*ii*) if $P = Q$, $R = 2P\cos(\theta/2)$

(*iii*) if $\theta = 0°$, $R = P - Q$

(*iv*) if $\theta = 180°$, $R = P + Q$

Ans. (*ii*) if $P = Q$, $R = 2P\cos(\theta/2)$

Q.1.(*b*) The bending equation is:

(*i*) $\frac{M}{I} = \frac{\sigma}{R} = \frac{E}{y}$

(*ii*) $\frac{M}{y} = \frac{\sigma}{I} = \frac{E}{R}$

(*iii*) $\frac{M}{y} = \frac{\sigma}{R} = \frac{E}{I}$

(*iv*) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

Ans. (*iv*) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

Q.1. (*c*) Moment of inertia of a circular area, about an axis perpendicular to the area passing through its centre is given by:

(*i*) $\pi d^4/8$ (*ii*) $\pi d^4/16$

(*iii*) $\pi d^4/32$ (*iv*) $\pi d^4/64$

Ans. (*iii*) $\pi d^4/32$

Q.1. (*d*) In UDL loading (wN/m), the maximum bending moment in case of simple supported beam is given as:

(*i*) wL (*ii*) $\frac{wL^2}{2}$

(*iii*) $\frac{wL^2}{4}$ (*iv*) $\frac{wL^2}{8}$

Ans. (*iv*) $\frac{wL^2}{8}$

Fill in the blanks for the following three parts:

You will be awarded full marks, if all the entries in a part are correct, otherwise will be awarded zero.

Q.1. (*e*) In a cantilever beam carrying a concentrated load at the free end, the bending moment will be maximum at .... and minimum at ....

Ans. Fixed end, Free end

Q.1. (*f*) A body will be in rotational equilibrium if resultant force .... and resultant moment ....

Ans. is zero or pass through centre of rotation is zero.

Q.1. (*g*) Theory of simple bending assumes the material of beam to be .... and perfectly ....

Ans. Homogeneous, Isotropic

Match the column for the following three parts: *You will be awarded full marks, if all the matches in a part are correct otherwise will be awarded zero.*

Q.1. (*h*) Match the following columns:

Column I	Column II
(*i*) Principle of superposition	(*P*) Dynamic equilibrium of particle
(*ii*) Maxwell theorem	(*Q*) Principle of moments
(*iii*) D'Alembert's principle	(*R*) Sum of deformation in individual section
(*iv*) Varignon's theorem	(*S*) Force analysis of trusses

Ans. (*i*) → *R*

(*ii*) → *S*

(*iii*) → *P*

(*iv*) → *Q*

(*i*) Column II gives the mass moment of inertia of various solids about the central axis. Match the following columns:

Column I	Column II
(*i*) Cylinder	(*P*) $2mr^2/3$
(*ii*) Sphere	(*Q*) $3mr^2/10$
(*iii*) Cone	(*R*) $2mr^2/5$
(*iv*) Thin spherical shell	(*P*) $mr^2/2$

Ans. (*i*) → *S*

(*ii*) → *R*

(*iii*) → *Q*

(*iv*) → *P*

(*j*) Match the following dimensional formula:

Column I	Column II
(*i*) Work	(*P*) $ML^{-1}T$
(*ii*) Power	(*Q*) ML^2T^2
(*iii*) Momentum	(*R*) ML^2T^3
(*iv*) Pressure	(*S*) MLT^{-1}

Ans. (*i*) Work → (*Q*) ML^2T^2

(*ii*) Power → (*R*) ML^2T^{-3}

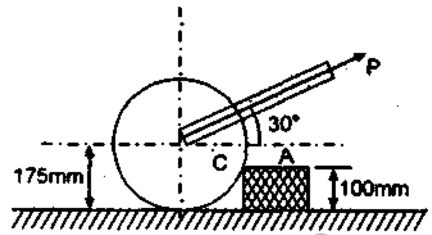
(*iii*) Momentum → (*S*) MLT^{-1}

(*iv*) Pressure → (*P*) $ML^{-1}T^{-2}$

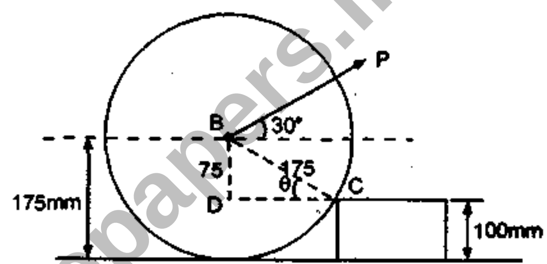
SECTION-B

2. Answer any three parts of the following: (10×3 = 30)

Q. 2(*a*) A roller shown in figure 1 is of mass 20 kN. What force *P* is necessary to start the roller over the block *A* and reaction at the contact point *C*.



Ans.



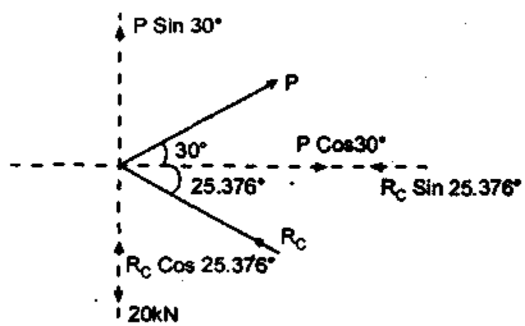
From the geometry,

$$\sin\theta = \frac{BD}{BC} = \frac{75}{175} = 0.42857$$

$$\theta = \sin^{-1} 0.42857$$

$$\theta = 25.376°$$

Free body diagram of the figure.



For equilibrium condition

$$\Sigma V = 0$$

$$P\sin 30° + R_c \sin 25.376 = 20$$

$$0.5\ P + 0.4285\ R_c = 20 \quad ...(i)$$

$$\Sigma H = 0$$

$$P\cos 30° = R_c\cos 25.376$$

$$0.866 = 0.9\ R_c$$

$$0.866\ P - 0.9\ R_c = 0 \quad ...(ii)$$

From equations (*i*) and (*ii*)

$R_c = 21.096 \quad N \quad$ Ans.

$P = 21.924 \quad N \quad$ Ans.

Q. 2(*b*) Calculate the values of shear force and bending moments for the cantilever beam shown in Fig. 2. Also draw the shear fore and bending moment diagrams.

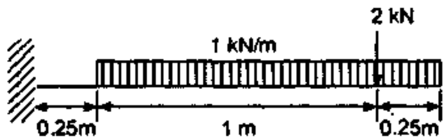
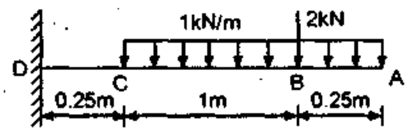


Fig. 2.

Ans.



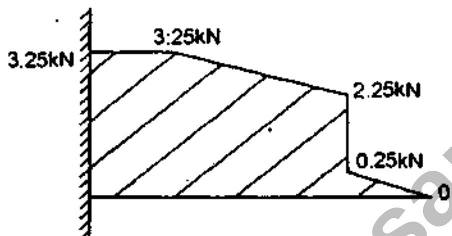


Fig. Shear force diagram

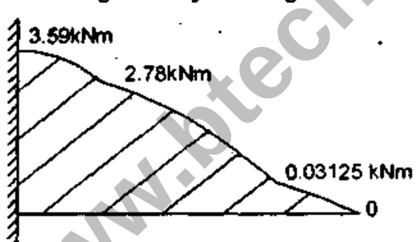


Fig. Bending moment diagram

S.F.D.

S.F. at point 'A' = 0

S.F. at Just Right point 'B'

$= 1 \times 0.25 = 0.25$ KN

S.F. at point 'B' = $0.25 \times 1 + 2 = 2.25$ kN

S.F. at point 'C' = $2.25 + 1 \times 1 = 3.25$ kN

S.F. at point 'D' = 3.25 kN

B.M.D.

B.M. at point 'A' = 0

B.M. at point 'B'

$= 1 \times 0.25 \times 0.125 = 0.03125$ kNM

B.M. at point 'C'

$= 1 \times 0.25 \times (1 + 0.125) + 2 \times 1 + 1 \times 1 \times 0.5$

$= 2.78$ kNM

B.M. at point 'D'

$= 1 \times 0.25 \times 1.375 + 2 \times 1.25 + 1 \times 1 \times .75$

$= 3.59$ kNM **Ans.**

Q. 2(*c*) Calculate the moment of inertia of the composite area shown in Fig. 3 about the centroidal axis.

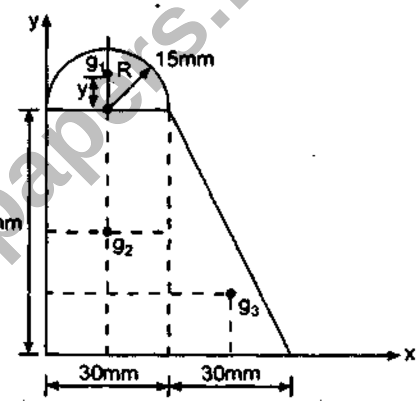


Fig. 3.

Ans. We know that Centroid of semi circle is

$$(\bar{Y}) = \frac{4R}{3\pi} = \frac{4 \times 15 \times 7}{3 \times 22}$$

$$\bar{Y}_1 = 6.36 \text{ mm}$$

$$\bar{X}_1 = 0$$

So, $g_1 = (0, 6.36)$

Centroid of rectangular,

$$\bar{X}_2 = \frac{30}{2} = 15 \text{ mm}$$

$$\bar{Y}_2 = \frac{40}{2} = 20 \text{ mm}$$

so, $g_2 = (15, 20)$

Centroid of triangle = $\bar{X}_3 = \frac{1}{3} \times 30 = 10$ mm

$$\bar{Y}_3 = \frac{1}{3} \times 40 = 13.33 \text{ mm}$$

$$g_3 = (10, 13.33)$$

Now, Area of semi-circle

$$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 15 \times 15$$

$A_1 = 353.57 \text{ mm}^2$

Area of rectangular

$= A_2 = a \times b = 40 \times 30$

$A_2 = 1200 \text{ mm}^2$

Area of triangle

$$A_3 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 30 \times 40$$

$= 600 \text{ mm}^2$

Now, $X = \dfrac{\text{Moment about } Y \text{ axis}}{\text{Total area}}$

$$= \frac{A_1\bar{X}_1 + A_2\bar{X}_2 + A_3\bar{X}_3}{A_1 + A_2 + A_3}$$

$$= \frac{353.57 \times (\bar{X}_1 + 15) + 1200 \times (15) + 600 \times (\bar{X}_3 + 30)}{353.57 + 1200 + 600}$$

$$= \frac{353.57 \times 15 + 1200 \times 15 + 600 \times 40}{2153.57}$$

$X = 21.965$ mm

$$\bar{Y} = \frac{\text{Moment about } \bar{X} \text{ axis}}{\text{Total area}}$$

$$= \frac{353.57 \times 46.36 + 1200 \times 20 + 600 \times 13.33}{2153.57}$$

$= 22.46$

Hence, $G = (X, Y)$ or $(21.96, 22.46)$

We know that moment of inertia of all sections about their centroid as,

Circle $I_{g_1} = \dfrac{\pi}{64} d^4 \times \dfrac{1}{2} = \dfrac{\pi}{64} \times (30)^4 \times \dfrac{1}{2}$

$I_{g_1} = 19888.39 \text{ mm}^4$

$I_{g_2} = \dfrac{bd^3}{12} = \dfrac{40 \times 30^3}{12}$

Rectangular

$I_{g_2} = 90.000 \text{ mm}^4$

Triangle $I_{g_3} = \dfrac{hb^3}{13} = \dfrac{40 \times 30^3}{36}$

$I_{g_3} = 30{,}000 \text{ mm}^4$

Now total moment of inertia of Composite structure is

$$I = I_{g_1} + A_1 Y_{c1}^2 + I_{g_2} + A_2 Y_{c2}^2 + I_{g_3} + A_3 Y_{c3}^2.$$

$= 19888.39 + 353.57 \times 23.9 + 90{,}000 + 1200 \times 2.46 + 30{,}000 + 600 \times 9.13$

$I = 156768.71 \text{ mm}^4$ **Ans.**

Q.2. (*d*) Two equal weights of 1000 *N* each are lying on two inclined planes connected by a string passing over a frictionless pulley as shown in Fig. 4. Using D'Alemberts principle, find the acceleration of weights and tension in the string. The coefficient of friction between the plane and weights is 0.2.

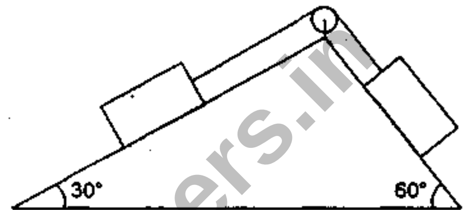
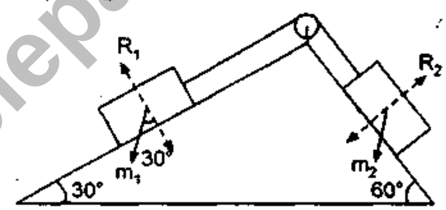


Fig.4.

Ans. $m_1 = m_2 = 1000\ N$, $\mu = 0.2$



Let a = Acceleration of the system, and

T = Tension in the string.

Normal reaction

$R_1 = m_1 \cos 30° = 1000 \times \cos 30°$

$R_1 = 866\ N$

Normal reaction

$R_2 = m_2 \cos 60° = 1000 + \cos 60°$

$R_2 = 500\ N$

Friction force

$(F_1) = \mu\ R_1 = 0.2 \times 866 = 173.2\ N$

Friction force

$(F_2) = \mu\ R_2 = 0.2 \times 500 = 100\ N$

From the figure we can see that the mass m_2 comes down while mass m_1 goes up.

So, total resultant force on mass (m_1)

$P = T - m \sin 30° - 173.2$

$P = T - 1000 \times \sin 30° - 1732$

$P_1 = T - 673.2$ ...(1)

Total resultant force on mass (m_2)

$P = m_2 \sin 60° - T - 100$

$P = 1000° \times \sin 60° - T - 100$

$P_2 = 766 - T$ ...(2)

We know that force

$$F = ma$$

so. $P_1 = m_1 a$

and $P_2 = m_2 a$

From equations (1) and (2)

$$m_1 a = T - 673.2$$

$$1000\, a = T - 673.2 \quad ...(3)$$

$$1000\, a = 766 - T \quad ...(4)$$

For solving equations (3) and (4)

$a = 0.0464$ m/s^2 **Ans.**

$T = 719.6\ N$ **Ans.**

Q 2.(e) Three beams have the same length, same allowable stress and the same bending moment. The cross-sections of the beam are square, a rectangle with depth twice the width, and a circle. Find the ratios of weights of rectangular and circular cross-section beams with respect to the square beam.

Ans. Both the beams are made of the same material and accordingly bending stress is same for both the sections. Then from the relation of pure bending.

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma . I}{y} = \sigma . Z$$

$$M = \sigma Z$$

(where Z = Section modulus)

the strength of two sections differ by this section modulus.

Now consider that side of the square is a then,

section modulus of square $(Z_s) = \frac{a^3}{6}$

Consider diameter of circle is d. then section modulus of circle

$$(Z_C) = \frac{\pi}{32} d^3$$

Consider width of the rectangular is r, and depth S, then section modulus of the rectangular

$$(Z_R) = \frac{rs^2}{6}$$

According to the question $S = 2r$

So, $Z_R = \frac{r.(2r)^2}{6} = \frac{8}{6} r^3$

$$Z_R = \frac{4}{3} r^3$$

because beams have equal bending stresses,

So, $M_S = M_C = M_R$ and $\sigma_S = \sigma_C = \sigma_R$

So, $M_S = M_C = M_R$ and $Z_S = Z_C = Z_R$

First consider

$$M_S = M_C \text{ or } Z_S = Z_C$$

$$\frac{a^3}{6} = \frac{\pi}{32} d^3$$

$$\frac{a^3}{d^3} = \frac{\pi \times 6}{32} = 0.589$$

$$\frac{a}{d} = 0.838 \quad ...(i)$$

Second consider $Z_S = Z_R$

$$\frac{a^3}{6} = \frac{4}{3} r^3$$

$$\frac{a^3}{r^3} = \frac{4 \times 6}{3} = 8$$

$$\frac{a}{r} = 2$$

because length of the beams are same,

So volume of square $= a^2 \times L$

Volume of circle

$$= \frac{\pi}{4} d^2 \times L = \frac{\pi}{4} \times \left(\frac{a}{.838}\right)^2 \times L$$

$$= 1.11\ a^2 \times L$$

Volume of rectangular = r.s.L = r.2r.L = 2 r^{2}L

$$= 2 \times \left(\frac{a}{2}\right)^2 \times L = \frac{1}{2} \times a^2 \times L$$

Consider density of the material is same, then ratio of beam weight.

$$W_S : W_C : W_R = 1 : 1.11 : \frac{1}{2}$$

or $\boxed{W_S : W_C : W_R = 1 : 1.11 : 0.5}$

SECTION-C

3. Answer any one of the following:

Q.3. (a) Explain the following:

(i) Laws of friction

(ii) Varignon's theorem

(iii) Types of force systems

Ans. (*i*) Laws of friction: Following are the laws of friction:

1. The force of friction acts in the opposite direction in which surface is having tandency to move.
2. The force of friction is equal to the force applied to the surface, so long as the surface is at rest.
3. When the surface is on the point of motion, the force of friction is maximum and this maximum frictional force is called limiting frictional force.
4. The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.
5. The limiting frictional force does not depend upon the shape and areas of the surfaces in contact.
6. The force of friction is independent velocity of sliding.

(*ii*) Varignon's theorem: The moment about a given point O of the resultant of several concurrent forces is equal to the sum of the moments of the particular forces about the same point. This is

$r \times F_1 + r \times F_2 = r \times (F_1 + F_2)$

The theorem follows from distributive property of vector product.

(*iii*) Types of force systems: A force system may be coplanar or non-coplanar. Hence, a force system is classified as:

(*a*) Coplanar
- Collinear
- Concurrent
- Parallel
- Non-concurrent

(*b*) Non-coplanar
- Concurrent
- Parallel
- Non-concurrent-Non-coplanar

Q.3. (*b*) What is a free body diagram? Explain with suitable example. Two smooth balls each of radius 15 cm and weighing 400 *N* are lying in a vertical cylinder of diameter 50 cm (Figure 5). Determine the pressure exerted on the balls and base of the cylinder by the balls.

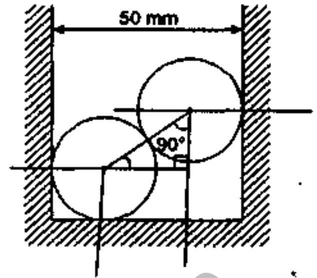
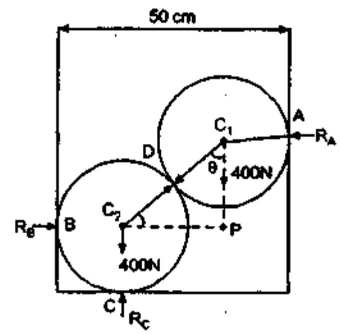


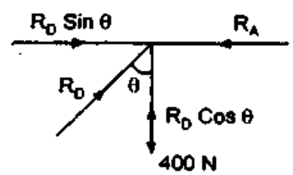
Fig. 5.

Ans. A free body diagram, also called the force diagram. A free body diagram shows all forces of all types acting on this body. Drawing such a diagram can aid in solving for the unknown forces or the equations of motion of the body creating a free body diagram can make it easier to understand the forces and torque in relation to one another and suggest the proper concepts to apply in order to find the solution to a problem for example shear force and bending moments in beams.

Numerical part:



Draw the free body diagram of ball C_1



For equilibrium conditions.

$$\Sigma H = 0$$

$$R_D \sin\theta = R_A$$

$$0.666\, R_D - R_A = 0 \quad ...(i)$$

$$\Sigma V = 0$$

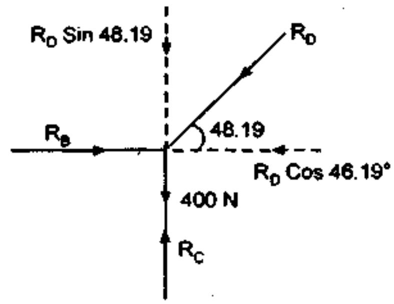
$$R_D \cos\theta = 400$$

$$R_D = \frac{400}{\cos 41.81} = 536.65 \text{ N} \quad \textbf{Ans.}$$

From equation (*i*)

$R_A = 536.65 \times 0.666 = 357.41\ N$ **Ans.**

Draw free body diagram of ball C_2



From the geometry

$C_2P = 20$ cm

$C_1C_2 = 30$ cm

$$\sin\theta = \frac{C_2P}{C_1\,C_2} = \frac{20}{30}$$

$\theta = 41.81°$

For equilibrium conditions

$\Sigma H = 0$

$R_B = R_D \cos 48.19°$

$= 536.65 \times \cos 48.19°$

$R_B = 357.41\ N$ **Ans.**

$\Sigma V = 0$

$R_C = 400 + R_D \times \sin 48.19°$

$= 400 + (399.99 \approx 400)$

$R_C = 800\ N$ **Ans.**

4. Answer any one of the following: (10)

Q. 4(*a*) A truss is loaded as shown in Fig. 6. Find the reactions and forces in the members of truss.

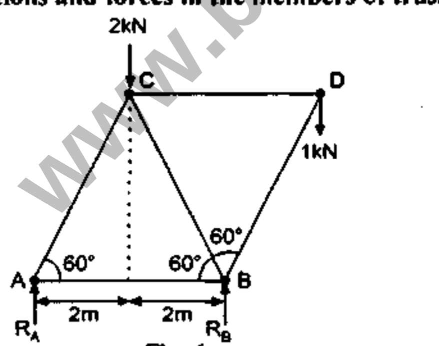


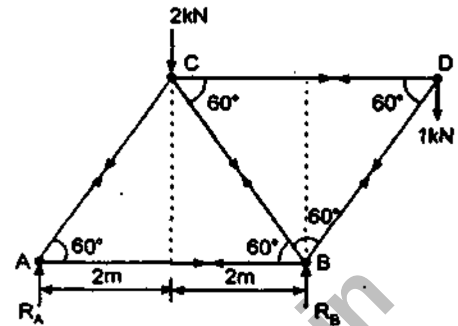
Fig. 6.

Ans. For equilibrium condition

$\Sigma V = 0$

$R_A + R_B = 2 + 1$

$R_A + R_B = 3$ ...(*i*)



Taking moment about point '*A*'

$\Sigma M_A = 0$

$1 \times 6 - R_B \times 4 + 2 \times 2 = 0$

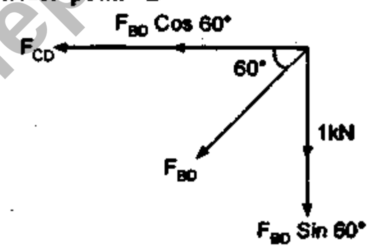
$R_B = 2.5$ kN **Ans.**

From equation (*i*)

$R_A = 3 - 2.5$

$R_A = 0.5$ kN **Ans.**

Consider point 'D'



$\Sigma V = 0,\ F_{BD} \sin 60° = -1$

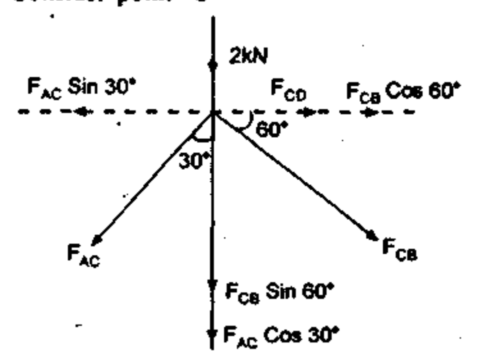
$F_{BD} = -1.155$ kN (Comp.) **Ans.**

$\Sigma H = 0$

$F_{CD} = -F_{BD} \cos 60° = 1.155 \times \cos 60°$

$F_{CD} = 0.577$ kN (Tensile) **Ans.**

Consider point 'C'



$\Sigma V = 0$

$F_{CB} \sin 60° + F_{AC} \cos 30° = -2$

$0.866\ F_{CB} + 0.866\ F_{AC} = -2$ ...(1)

$\Sigma H = 0$

$F_{AC} \sin 30° - F_{CB} \cos 60° = F_{CD}$

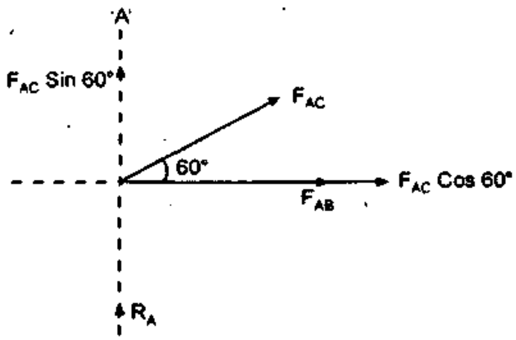
$0.5\ F_{AC} - 0.5\ F_{CB} = 0.577$ ...(ii)

From equations (i) and (ii)

$F_{AC} = -\ 0.577$ kN (comp.) **Ans.**

$F_{CB} = -\ 1.731$ kN (comp.) **Ans.**

Consider point 'A'



$\Sigma H = 0$

$F_{AB} = -\ F_{AC} \times \cos 60°$

$F_{AB} = +\ 0.577 \times \cos 60°$

$F_{AB} = 0.2885$ kN (Tensile) **Ans.**

Q. 4(b) Draw the shear force and bending moment diagrams for the simple supported beam as shown in Fig. 7.

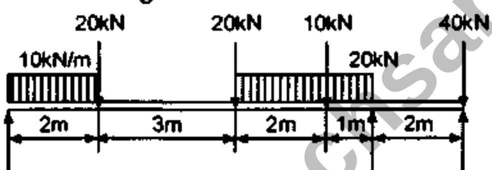
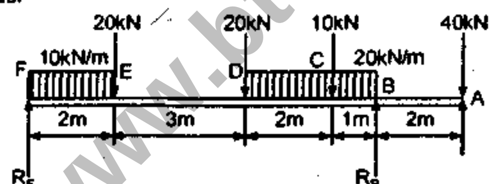


Fig. 7.

Ans.



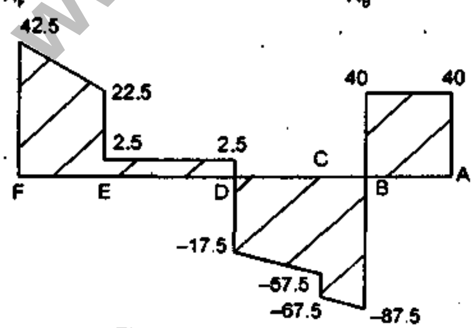


Fig. Shear force diagram

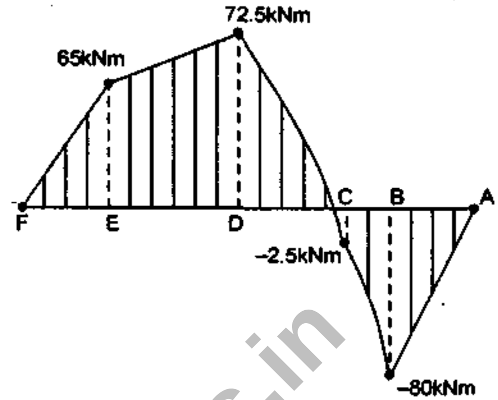


Fig. Bending moment diagram

For equilibrium conditions

$\Sigma V = 0$

$R_B + R_F = 170$ ...(i)

Taking moment about point 'F'

$R_B \times 8 = 40 \times 10 + 20 \times 1 \times 7.5 + 10 \times 7 + 20 \times 2 \times 6 + 20 \times 5 + 20 \times 2 + 10 \times 2 \times 1$

$R_B = 127.5$ kN

From equation (i)

$R_F = 42.5$

Shear force diagram:

Shear force at point 'F' = 42.5 kN

Shear force at just left of point 'E'

= 42.5 − 10 × 2 = 22.5 kN

Shear force at point 'E'

= 42.5 − 10 × 2 − 20 = 2.5 kN

Shear force at just left of point 'D' = 2.5 kN

Shear force at point 'D'

= 2.5 − 20 = − 17.5 kN

Shear force at just left of point 'C'

= − 17.5 − 20 × 2 = − 57.5 kN

Shear force at point 'C'

= − 57.5 − 10 = − 67.5 kN

Shear force at just left of point 'B'

= − 67.5 − 20 × 1 = − 87.5 kN

Shear force at point 'B'

= − 87.5 + (R_B = 127.5) = 40 kN

Shear force at point 'A' = 40 − 40 = 0

Bending moment diagram:

Bending moment at point 'F' = 0

Bending moment at point 'E'

= 42.5 × 2 − 10 × 2 × 1 = 65 kNm

Bending moment at point 'D'

$= 42.5 \times 5 - 10 \times 2 \times 4 - 20 \times 3$
$= 72.5$ kNm

Bending moment of point 'C'

$= 42.5 \times 7 - 10 \times 2 \times 6$
$- 20 \times 5 - 20 \times 2 - 20 \times 2 \times 1$
$= - 2.5$ kNm

Bending moment at point 'B'

$= 42.5 \times 8 - 10 \times 2 \times 7 - 20 \times 6$
$- 20 \times 3 - 20 \times 3 \times 2 - 10 \times 1$
$- 20 \times 1 \times .5 = - 80$ kNm

Bending moment at point 'A'

$= 42.5 \times 10 - 10 \times 2 \times 9 - 20 \times 8$
$- 20 \times 5 - 20 \times 2$ [illegible] $\times 4 - 10 \times 3 - 20 \times 1 \times 2.5$
$+ 127.5 \times 2 = 0$

5. Answer any one of the following:

Q. 5(*a*) (*i*) State and prove the theorems of parallel and perpendicular axis with suitable example.

Ans. Theorem of perpendicular axis: It states that, if I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at 0, the moment of inertia I_{ZZ} about the axis $Z - Z$ perpendicular to the plane and passing through the intersection of $X - X$ and $Y - Y$ is given by

$$I_{ZZ} = I_{XX} + I_{YY}$$

Proof: Consider a small lamina (P) of area a having co-ordinates as x and y along OX and OY two mutually perpendicular axes on a plane section as shown in figure (i)

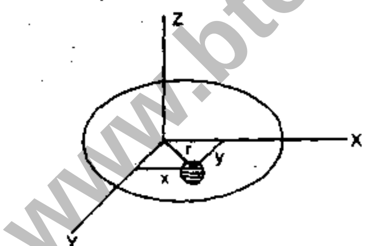


Fig. (i)

From the geometry of figure, we find that

$$r^2 = x^2 + y^2$$

We know that the moment of inertia of the lamina P about XX-axis,

$$I_{XX} = \text{da} \cdot y_2$$

Similarly $I_{YY} = \text{da} \cdot x^2$

and $I_{ZZ} = \text{da}. r^2 = \text{da}\,(x^2 + y^2)$

$= \text{da} \cdot x^2 + \text{da} \cdot y^2$

$$I_{ZZ} = I_{XX} + I_{YY}$$

Theorem of parallel axis: It states, if the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G. Then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by:

$$I_{AB} = I_G + ah^2$$

Proof: Consider a strip of a circle, whose moment of inertia is required to be found out about a line AB as shown in figure (ii)

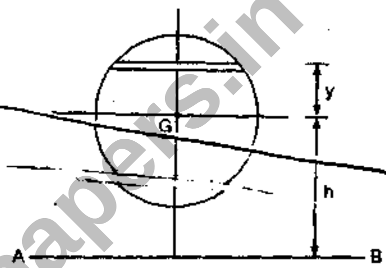


Fig. (ii)

We know that moment of inertia of the whole section about an axis passing through the centre of gravity of the section.

$$= \delta a \,.\, y^2$$

and moment of inertia of the whole section about an axis passing through its centre of gravity.

$$I_G = \Sigma\, \delta a \,.\, y^2$$

$\therefore$ Moments of inertia of the section about the axis AB,

$$I_{AB} = \Sigma\, \delta a\,(h + y)^2$$
$$= \Sigma\, \delta a\,(h^2 + y^2 + 2\,hy)$$
$$= \Sigma\, \delta a.h^2 + \Sigma\, \delta a.y^2 + \Sigma\, \delta a.2hy$$
$$I_{AB} = ah^2 + IG$$

(*ii*) Derive an expression for the mass moment of inertia of a circular disc of radius R and thickness *t* about its centroidal axis.

Ans. Figure shows a thin circular plate of radius R and uniform thickness t. If p is the density of the plate material, then mass of the plate

M = density × volume

= density × (area × thickness) = $\rho\pi R^2 t$

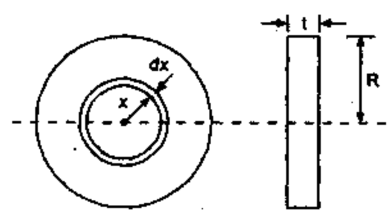


Fig. Circular disc

Consider an elementary ring of radius r and width dr.

Mass of elemental ring

$$dm = \rho[\pi (r + dr)^2 - \pi r^2]\, t$$
$$= \rho(2\pi r dr)\, t = 2\pi t\delta r dr$$

Mass moment of inertia of this elementary ring about the polar axis zz.

$$dmr^2 = 2\pi t\ \rho r^3 dr$$

Mass moment of inertia of the circular plate about polar axis ZZ.

$$2\pi\, t\, \rho \int_0^R r^3\, dr = 2\pi\, t\, \rho \left[\frac{r^4}{4}\right]_0^R$$

$$= \rho\pi R^2 t \times \frac{R^2}{2} = \frac{MR^2}{2}$$

Where $M = \delta\pi R^2 t$ is the mass of circular lamina. Invoking the theorem of perpendicular axis, the mass moment of inertia of a circular lamina about XX or YY axis is

$$I_{XX} = I_{YY} = \frac{I_{ZZ}}{2} = \frac{1}{4} MR^2$$

Q.5. (b) A semicircular area is removed from the trapezoid as shown in Fig. 8. Determine the centroid of the remaining area.

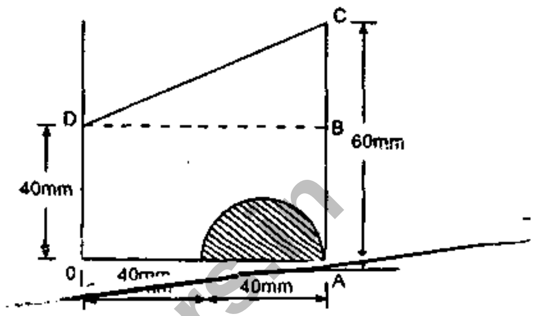


Fig. 8

Ans. Formula used for the semiconductor are:

$$\bar{x} = 0,\ \bar{y} = \frac{4r}{3\pi}$$

The area can be considered to contain a triangle and a rectangle with a semicircular portion removed from it.

To find the centroid.

Element	Area. A mm²	$\bar{X}$ mm	$\bar{Y}$ mm	$A\bar{X}$ mm³	$A\bar{Y}$ mm³
Triangle (DBC)	$\frac{1}{2} \times 20 \times 80 = 800$	$2\left(\frac{80}{3}\right) = 53.33$	$40 + \frac{20}{3} = 46.66$	= 42664	= 37328
Rectangle (OABD)	$40 \times 80 = 3200$	40	= 20	= 128000	= 64000
Semicircle	$-\frac{\pi (20)^2}{2} = -628.318$	40 + 20 = 60	$\frac{4}{3\pi}(20) = 8.48$	– 37699.08	= – 5328.13

$\Sigma A = 3371.68$

$\Sigma A\bar{X} = 132964.92$

$\Sigma A\bar{Y} = 95999.87$

$$\bar{X} = \frac{\Sigma A\bar{X}}{\Sigma A} = \frac{132964.92}{3371.68} = 39.43$$

$$\bar{Y} = \frac{\Sigma A\bar{Y}}{\Sigma A} = \frac{95998.87}{3371.68} = 28.47$$

Hence, centroid of structure is

G (39.43, 28.47) **Ans.**

6. Answer one of the following: (10)

Q.6. (a) (i) What do understand by the term kinematics? Explain different types of plane motion of rigid bodies with suitable examples.

Ans. Kinematics of motion: The relative motion of bodies without, consideration of the forces causing the motion. In other words, kinematics deal with the geometry of motion and concepts like displacement, velocity and acceleration considered as function of time.

Plane motion: When the motion of a body is confined to only one plane, the motion is said to be plane motion. The plane motion may be either rectilinear or curvilinear.

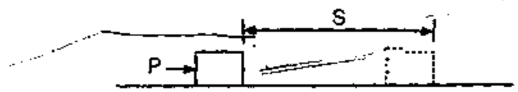
Rectilinear motion: It is the simplest type of motion and is along a straight line path. Such a motion is also known as transiatory motion.

Curvilinear motion: It is the motion along a curved path. Such a motion when confined to one plane, is called plane curvilinar motion.

Q.6. (*a*) (*ii*) What is energy? Explain the various forms of mechanical energies.

Ans. Energy: Whenever a force acts on a body and the body undergoes some displacement then work is said to be done that is called energy.

If a force P, acting on a body, causes it to move through a distance s, as shown in figure.



then, work done = Force × Distance = $P \times S$

If P makes some angle to the motion direction say θ. Then work done = $P\cos\theta \times S$.

Unit of Energy: The unit of energy depend upon the units of the force and distance.

One N-m, It is the work done by a force $1N$, when it displaces the body through 1m, It is called Joule.

1 Joule = 1 N-m

Various forms of mechanical energies:

Mechanical Energies are:

1. Kinetic energy
2. Potential energy
3. Impulse energy
4. Vibrational energy etc.

Q. 6(*b*) A bullet of mass 20 g is fired horizontally with a velocity of 300 m/sec, from a gun carried in a carriage, which together with the gun has a mass of 100 kg. The resistance to sliding of the carriage over the ice on which it rests is 20 N. Find:

(*i*) velocity, with which the gun will recoil

(*ii*) distance, in which it comes to rest, and

(*iii*) time taken to do so.

Ans. Given: $M = 20\ g = 20 \times 10^{-3}$ kg, m = 100kg

$v = 300$ m/sec $F_r = 20$ N

(*i*) Momentum of bullet = Momentum of carriage

$$mv = MV_1$$

$$20 \times 10^{-3} \times 300 = 100 \times V_1$$

$$V_1 = 0.06 \text{ m/s} \quad \text{Ans.}$$

(*ii*) Resistance of sliding = 20 N

$$F = Ma = 20$$

$$a = \frac{20}{100} = 0.2 \text{ m/s}$$

We know

$$V^2 = V_1^2 - 2as$$

$$0 = (0.06)^2 - 2 \times 0.2 \times S$$

$$S = 0.009 \text{ m or } 9 \text{ mm} \quad \text{Ans.}$$

(*iii*) $V = V_1 - at$

$$0 = 0.06 - 0.2 \times t$$

$$t = 0.3 \text{ sec.} \quad \text{Ans.}$$

7. Answer any one of the following: (10)

Q.7. (*a*) What do you understand by the term neutral axis and neutral surface? A rectangular beam 300 mm deep is simply supported over a span of 4 m. Determine the uniformly distributed load per meter, which the beam can carry, if the bending stress does not exceed 120 N/mm².

Take $I = 8 \times 10^6$ mm⁴

Ans. Neutral axis: The neutral axis is an axis in the cross section of a beam or shaft along which there are no longitudinal stresses or strains.

Neutral surface: In the process of bending there are axial lines that do not extend or conrtact. The surface described by the set of lines that do not extend or contract is called the neutral surface.

Given: $I = 8 \times 10^6$ mm^4, $\sigma = 120$ N/mm^2

= 300 mm, $l = 4$ m = 4000 mm

We know that

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}; \quad \frac{M}{8 \times 10^6} = \frac{120}{150}$$

$$(\because y = d/2 = 300/2 = 150 \text{ mm})$$

$$M = 6.4 \times 10^6 \text{ N-mm}$$

We know also that maximum bending moment of UDL on simply supported beam:

$$\frac{Wl^2}{8} = 6.4 \times 10^6; \quad W = \frac{6.4 \times 10^6 \times 8}{4000 \times 4000}$$

$$= 3.2 \text{ N/mm or} = 3200 \text{ N/m} \quad \textbf{Ans.}$$

Q. 7(*b*) State the assumptions made in the theory of pure torsion. Derive the torsion formula:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

Ans. The torsion equation is based on the following assumptions:

1. The material of the shaft is uniform throughout.
2. The shaft circular in section remains circular after loading.

3. A plane section of shaft normal to its axis before loading remain plane after the torque have been applied.
4. The twist along the length of shaft is uniform throughout.
5. The distance between any two normal cross-sectional remains the same after the application of torque.
6. Maximum shear stress induced in the shaft due to application of torque does not exceed its elastic limit value.

Let T = Maximum twisting torque
D = Diameter of the shaft.
J = Polar moment of inertia
t = Shear stress
C = Modulus of rigidity
θ = The angle of twist and
l = length of shaft

In figure (*i*) is shown a shaft fixed at one end and torque being applied at the other end. If a line LM is drawn on the shaft, it will be distorted to LM′ on the application of the torque, thus cross-section will be twisted through angle θ and surface by angle ϕ.

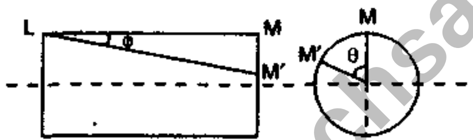


Fig. (i)

Here, shear strain,

$$\phi = \frac{MM'}{l}$$

Also, $$\phi = \frac{\tau}{c}$$

$$\frac{MM'}{l} = \frac{\tau}{c}$$

or $$\frac{R\theta}{l} = \frac{\tau}{c}$$

[$\because MM' = R \times \theta$, R being radius of shaft]

$$\therefore \quad \frac{\tau}{R} = \frac{C\theta}{l} \qquad ...(a)$$

Refer to figure (*ii*), consider an elementary ring of thickness dx at a radius x and let the shear stress at this radius be τ_x.

The turning force on elementary ring

$$= \tau_x \, . \, 2\pi x \, . \, dx$$

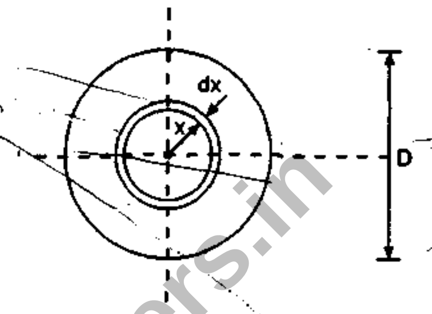


Fig. (ii)

Turning moment due to this turning force,

$$dT = \tau x \, . \, 2\pi x \times x$$

To get total turning moment,

$$\int dT = \int_0^R \tau_x . 2\pi x \, dx \, x$$

or $$\int dT = 2\pi \int_0^R \tau_x . x^2 . dx$$

$$= 2\pi \int_0^R \frac{\tau_x}{R} . x^2 dx \qquad \left[\because \frac{\tau}{R} = \frac{\tau_x}{x} \text{ or } \tau_x = \frac{\tau . x}{R}\right]$$

$$= 2\pi \frac{\tau}{R} \int_0^R x^3 \, dx = 2\pi \frac{\tau}{R}\left[\frac{x^4}{4}\right]_0^R$$

$$T = \tau . \frac{\pi R^3}{2} = \tau . \frac{\pi}{16} D^3$$

$$T = \frac{\tau}{R} . \frac{\pi R^4}{2} = \frac{\tau}{R} J$$

$$\frac{T}{J} = \frac{\tau}{R} \qquad ...(b)$$

From equations (*a*) and (*b*)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{l}$$ Proved.