B. TECH. EME102

FIRST SEMESTER EXAMINATION, 2010-11

ENGINEERING MECHANICS

Time : 3 Hours *Total Marks :* **100**

Note : (1) *Attempt all questions. Marks are indicated against each question.*
(2) *Assume missing data suitable, if any.*

SECTION - A

1. Attempt all parts:

Note: In parts (*i*) and (*ii*), choose the correct choice:

(*i*) A body of weight 100 N is resting on a rough horizontal table. The friction force acting on it is:

(*a*) 20 N
(*b*) 10 N
(*c*) 0 N
(*d*) The question cannot be answered without knowing the co-efficient of the friction.

Ans. (*c*)

(*ii*) First moment of area about an axis is zero. The axis

(*a*) Must be an axis of symmetry
(*b*) Must pass through CG
(*c*) Both (*a*) and (*b*)
(*d*) None

Ans. (*c*) Both (*a*) and (*c*)

Note: In part (*iii*) and (*iv*), choose whether the statement is true or false:

(*iii*) When nonconcurrent coplanar force act a body, it is possible that the resultant force is zero even if the body is not in equilibrium.

Ans. True

(*iv*) A solid shaft is stronger compared to a hollow shaft if material, weight length of the shafts are same.

Ans. False

Note: In part (*v*) and (*vi*), fill in the blanks: You will be awarded full marks if all the entries in pair are correct otherwise will be awarded zero.

(*v*) A perfect truss has minimum ____ joints and ____ members.

Ans. A perfect truss has minimum 3 joints and 3 members

(*vi*) The bending stress at neutral axis is ___ and at the top layer is ____

Ans. The bending stress at neutral axis is zero and at the top layer is maximum.

Note: In parts (*vii*) and (*viii*), matching, types:

You will be awarded full marks, if all the matches correct otherwise will be awarded zero.

(*vii*) Match the following columns:

Column I	Column 2
(*a*) Moment of inertia of a circular plate of mass *M* and radius *R* about its axis is	(*a*) $2MR^2/5$
(*b*) Moment of inertia of a circular ring of mass *M* and radius *R* about its axis is	(*b*) $MR^2/2$
(*c*) Moment of Inertia of solid sphere of mass *M* and radius *R* about its diameter is	(*c*) $2MR^2/3$
(*d*) Moment of Inertia of thin spherical shell of mass *M* and radius *R* about its diameter is	(*d*) MR^2

Ans. (a) – (2)
(b) – (4)
(c) – (1)
(d) – (3)

(viii) Match the following columns:–

Column 1	Column 2
(a) In pure rotation of rigid body, the radial component of acceleration is	(a) $R\alpha$
(b) In pure rotation of rigid body, tangential component of acceleration is	(b) $\frac{d^2s}{d^2}$
(c) In curvilinear motion of rigid body, the tangential component of acceleration is	(c) $\frac{1}{2}\left(\frac{ds}{dt^2}\right)^2$
(d) In curvilinear motion of rigid body, normal component of acceleration is	(d) $R\omega^2$

Ans. (a) – (4)
(b) – (1)
(c) – (2)
(d) – (3)

Note: In questions (ix) and (x), two statements are given followed by four choices. Choose the correct choice:

(ix) Statement 1: A moment of 10 N-m of applied in the middle of the simple supported beam. The magnitude of bending moment in the beam at the middle is 5 Nm.

Statement 2: A cantilever beam of length 2 m carries 10 kN force at a distance of 1 m form the support. The bending moment at a distance of 1.5 m from the support is 15 kNm.

(a) Only statement 1 is true
(b) Only statement 2 is true
(c) Both statements are true
(d) Neither statement is true

Ans. Only statement (1) is correct (a)

(x) Statement 1: A wire of length L is bent, the CG will remain at the middle.

Statement 2: From a solid circular plate of radius R a concentric circular plate of radius R/2 is removed. CG will remain unchanged.

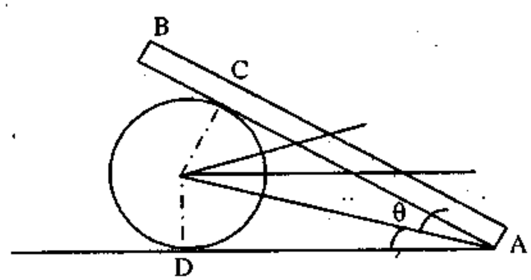
(a) Only statement 1 is true
(b) Only statement 2 is true
(c) Both statements are true
(d) Neither statement is true

Ans. Only statement 2 is true

SECTION -B

2. Answer any three parts of the following: (10 × 2 = 30)

(a) A smooth weightless cylinder of radius 600 mm rests on a horizontal plane and is kept from rolling by an inclined string of length 1000 mm. A bar AB of length 1500 mm and weight 1200 N is hinged at A and placed against the cylinder of negligible weight. Determine tension in the string.

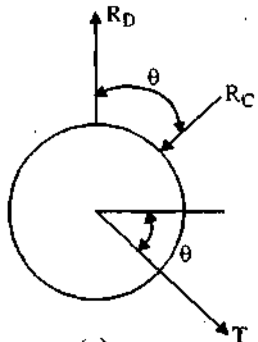


Ans. From fig. (c) $\sum M_A = 0$

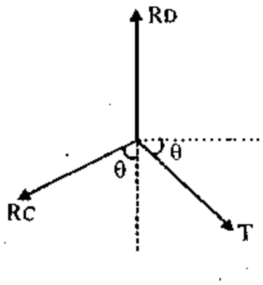
Taking moment about pint A, we get

$R_C \times 800 - 1200 \times 750 \cos 73.72 = 0$

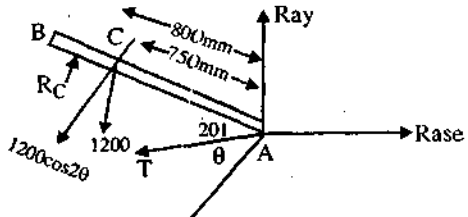
$$R_C = \frac{1200 \times 750 \cos 73.72}{800}$$



(a)



(b)



(c)

Free body diagram

$R_C = 315.37$ N

$$\sin\theta = \frac{600}{1000}$$

$\sin\theta = 0.6$

$\theta = 36.86$

$2\theta = 73.72$

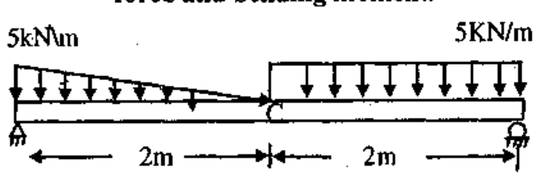
By lami's Theorem from fig (*b*)

$$\frac{R_C}{\sin(90+\theta)} = \frac{T}{\sin(180-2\theta)}$$

$$T = \frac{315.37 \times \sin(180-73.72)}{\sin(90+36.86)}$$

$T = 377.86$ N **Ans.**

(*b*) For the simply supported beam as shown in figure, draw shear force and bending moment diagrams after finding the equations for shear force and bending moment.



Ans. Total load

$R_A + R_B = 5 \times 2 + 5$

$= 10 + 5 = 15$ kN

$\Sigma M_A = 0$

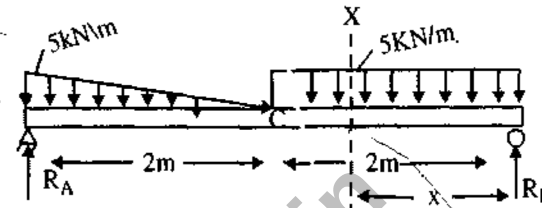


Fig (a)

Taking moment about point A we get,

$$5 \times \frac{2}{3} + 10 \times 3 = 4\ R_B$$

$$\frac{10}{3} + 30 = 4\ R_B$$

$$R_B = \frac{100}{3 \times 4} = \frac{25}{3} = 8.33 \text{ kN}$$

$R_B = 8.33$ kN

$R_A = 15 - 8.33$

$R_A = 6.67$ kN

For shear force diagram (*SFD*) load at point A

$SF_A = +R_A = 6.67$ kN

$SF_B = -8.33 + 5x \quad 0 \le x \le 2$

at distance 0

$= -8.33 + 5 \times 0$

$= -8.33$ kN

at distance 2.

$= -8.33 + 5 \times 2$

$= -8.33 + 10$

$= -1.67$ kN

with the help of load, draw the SFD shown in fig. (*b*)

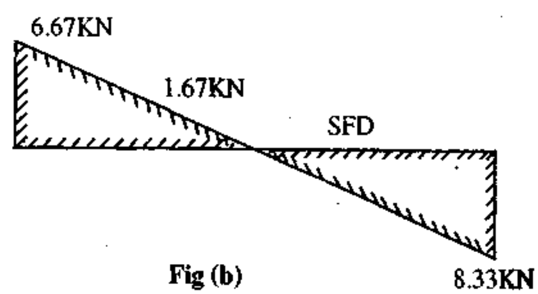


Fig (b)

For BMD- when the Beam is simply supported at end point B.M. will be zero.

$$BM_A = BM_B = 0$$

$$M = 8.33\,x - \frac{5}{2}\,x^2 \quad 0 \le x \le 2$$

→ at distance 2

$$8.33 \times 2\ \frac{5}{2} \times 42 = 6.66 \text{ kN.m}$$

$$= 8.33\,x - 10\,(x-1) - 1 - 25(x-2)2 \times \frac{1}{3}(x-2) \qquad 2 \le x \le 4$$

→ at distance 2

$$= 8.33 \times 2 - 10\,(2-1) - 1.25\,(2-2)^2 \times \frac{1}{3}\,(2-2)$$

$$= 16.66 - 10$$

$$= 6.66 \text{ kNM}$$

→ at distance 4 will be zero

with help of BM load we will draw the BMD show in fig (*c*).

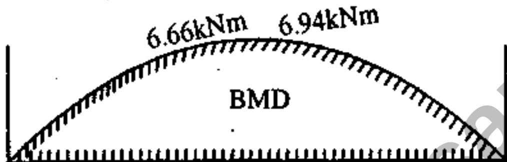
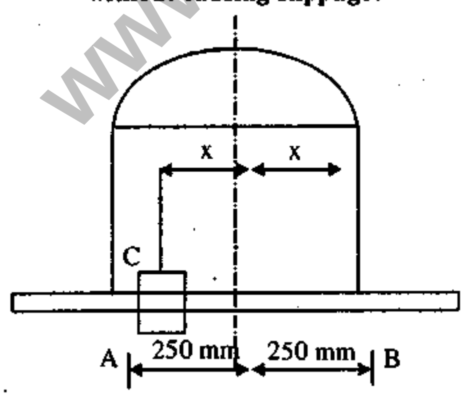


Fig (c)

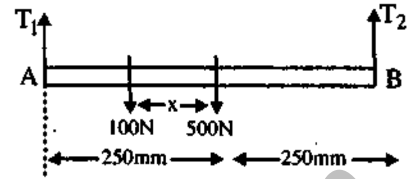
(*c*) **Rod *AB* of weight 500 N supported by a cable wrapped around a semi-cylinder having coefficient of friction of 0.2 A weight *C* weighing 100 N can slide without friction on rod AB. What is the maximum range *x* form centerline the mass *C* can be placed without causing slippage?**



Ans. From the fig.

$$T_1 + T_2 = 500 + 100$$

$$T_1 + T_2 = 600 \qquad \text{... (1)}$$



We know that

$$\frac{T_1}{T_2} = e^{+\mu\theta}$$

$$\frac{T_1}{T_2} = e^{0.2 \times \pi}$$

$$T_1 = 1.874\,T_2$$

Put the value of T_1 in equation (1) we have

$$1.874\,T_2 + T_2 = 600$$

$$2.8\,T_2 = 600$$

$$T_2 = 208.774$$

$$T_1 = 600 - 2087$$

$$T_1 = 391.23 \text{ M}$$

For the maximum range x

$$\Sigma M_A = 0$$

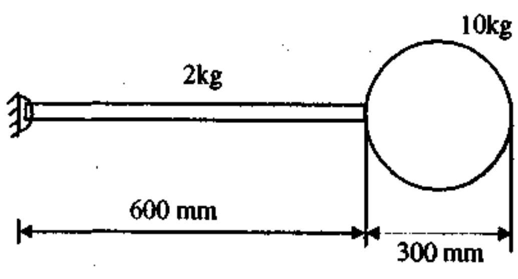
taking moment about point A we have

$$100 \times (250 - x) + 500 \times 250 = 208.37 \times 500$$

$$2500 - 100\,x + 125000 = 104165$$

$$x = 456.15 \text{ mm} \quad \textbf{Ans.}$$

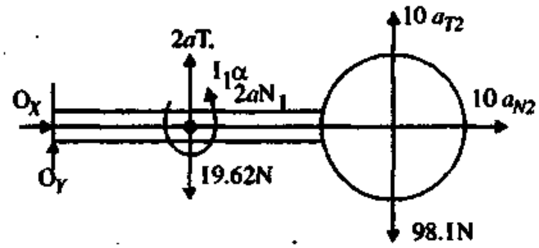
(*d*) **A homogeneous sphere weighing 10 kg is attached to a slender rod of mass 2 kg. If the system is released from horizontal position in rest condition, find the magnitude of angular acceleration. Also find angnlar velocity of system when it passes through vertical position.**



Ans. Given data

weight of sphere = 10 kg

weight of rod =2 kg



a_{T_1} (rod) = 0.3 α

a_{T_2} (sphere) = 0.75 α

$$I_G\text{ (rod)} = \frac{ml^2}{12} = \frac{2\times 0.6^2}{12} = 0.06 \text{ kg m}^2$$

$$I_G\text{ (shere)} = \frac{2}{5}mr^2 = \frac{2}{5}\times 10\times(0.15)^2$$
$$= 0.09 \text{ kg m}^2$$

$\Sigma M_0 = 0$

$= -(19.62\times 0.3) + (2a_{T_1}\times 0.3) + I_1\alpha - (98.1 \times 0.75) + (10\ a_{T_2}\times 0.75) + I_2\alpha = 0$

$= -5.889 + 2\times 0.3\,\alpha\times 0.3 + 0.06\,\alpha - (98.1\times 0.75) + (10\times 0.75\,\alpha\times 0.75) + 0.06\times\alpha = 0$

$= -5.889 + 0.18\alpha + 0.06\,\alpha - 73.575 + 5.625\alpha + 0.06\alpha = 0$

$\alpha = 13.34 \text{ rad/sec}^2$

Work done by gravity = mgh (sphere) + mgh (rod)

$= 10\times 9.81\times 900 + 2\times 9.81\times 600$

$= 79.456$ J

$KE_1 = 0,\ KE_2 = \frac{1}{2}I_0\omega^2$

$I_0 = (I_G + mk^2)\text{ rod} + (I_G + mk^2)\text{ sphere}$

$= 0.06 + 2x + 0.06 + 10x$

$= 5.955 \text{ kg m}^2$

$V_{1-2} = KE_2 - KE_1$

$79.456 = \frac{1}{2}\times 5.955\ \omega^2$

$\omega = 5.166$ rad/sec

(e) (i) **What do you understand by "Pure Bending**

(ii) **Determine the suitable values for inside and outside diameters of hollow steel shaft whose Internal diameter is 0.6 times its external diameter. The shaft transmits 220 kW at 200 rpm. The allowable shear stress is limited to 75 MPa, and angle of twist is limited to 1° per meter. the modulus of rigidity for shaft material is 80 kN/mm².**

Ans. (i) Pure Bending: Pure bending refers to loading of beam fashion that the beam is absolutely free from shear force and is subjected only to constant bending moment.

Ans. (ii) Given data $d_i = 0.6\ d_0$

$P = 220$ kw, N = 200 rpm

$\tau = 75$ MPa, $\frac{\theta}{l} = \frac{\pi}{180}$

$G = 80\times 10^9\ \text{N/m}^2$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{200\times 60\times 100}{2\pi\times 200}$$

$T = 10504.2$ NM.

$$J = \frac{\pi}{32}\ (d_o^4 - d_i^4)$$
$$= \frac{\pi}{32}\ (d_o^4 - 0.1296\ d_o^4)$$
$$= 0.08545\ d_o^4$$

We know that

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\Rightarrow \frac{10504.2}{0.08545\ d_0^4} = \frac{75\times 10^6\times 2}{d_0}$$

$$d_0^3 = \frac{10504.2}{150\times 0.0845\times 10^6}$$

$$d_0^4 = 9.358 \times 10^{-2}\ m = 93.58 \text{ mm}$$

$$\frac{T}{J} = \frac{G\theta}{l} \Rightarrow \frac{10504.2}{0.0854 \times 10^4} = 80 \times 10^9 \times \frac{\pi}{180 \times 1}$$

$$d_0^4 = \frac{10504.2 \times 180}{0.08545 \times 80 \times \pi \times 10^9}$$

$$d_0 = 9.86 \times 10^{-2}\ m = 96.86 \text{ mm}$$

Safe $d_0 = 96.86$ mm

$d_i = 96.8 \times 0.6 = 58.12$ mm **Ans.**

SECTION C

3. Attempt any one part of the following:

(*a*) (*i*) States and prove Lami's theorem.

Ans. (*i*) State and prove Lami's Theorem: It state that "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two" mathematically

$$\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$$

Proof: Draw the three force P_1, P_2 and P_3

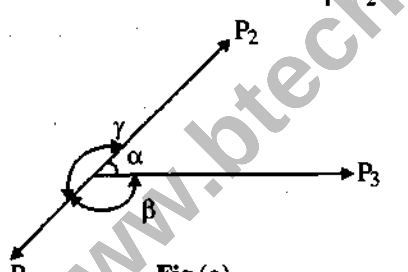


Fig (a)

are after the other indirection and magnitude starting from point. *a*. Since the body is in equilibrium (resultant is zero) the last point must coincide with a thus, it results in a tringle of force *abc* as shown in fig (*b*) Now, the external angle at *a*, *b* and *c* are equal to β, γ and α.

Since *ab* parallel to P_1

bc parallel to P_2 and

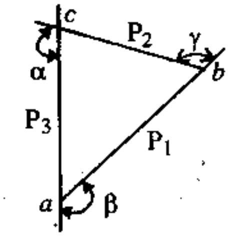


Fig (b)

ca parallel to P_3

in the tringle of forces *abc*

$ab = P_1$

$bc = P_2$ and

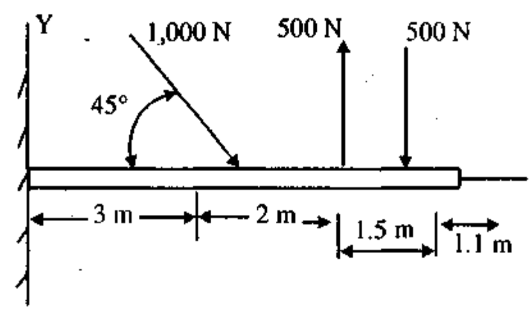
$ca = P_3$,

Applying sine rule for the tringle *abc*,

$$\frac{ab}{\sin(180-\alpha)} = \frac{bc}{\sin(180-\beta)} = \frac{ca}{\sin(160-\gamma)}$$

i.e., $\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$ **Proved**

(*ii*) A system of forces acting on a cantilever beam is shown in figure. Reduce this system to a single force system and find the point of application of this force on the beam.



Ans. (*ii*) For single inclined forces resolved vertically and horizontally find the value of vertical and horizontal force which is act the point

$$Fx = 1000 \cos 45°$$
$$= 1000 \times 0.707$$

$= 707.106$ N

$Fy = 1000 \sin 45$

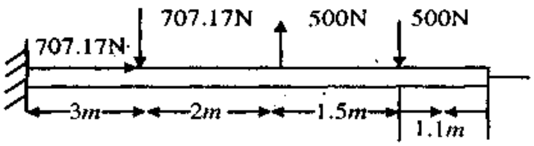
$= 1000 \times 0.707$

$= 707.106$ N

$\Sigma Fx = 707.17$ kN (→)

$\Sigma Fy = 707.17$ kN $- 500 + 500$

$\Sigma Fy = 707.17$ kN (↓)



Taking moment about fixed point

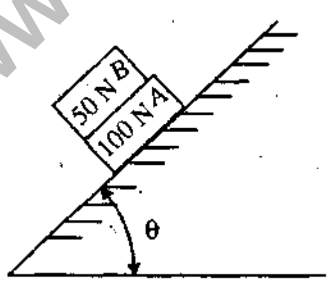
$\Sigma M = 707.17 \times 3 - 500 \times 5 + 500 \times 6.5$

$= 707.17x$

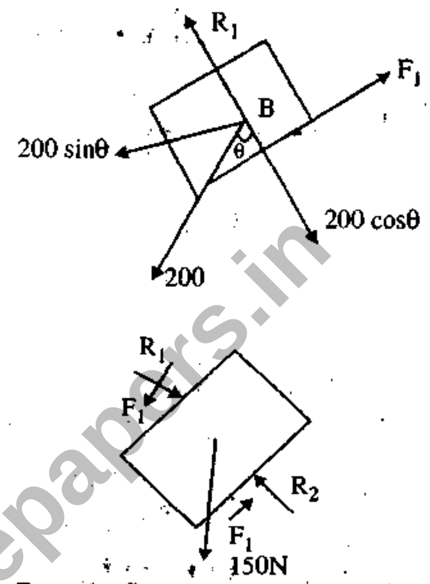
$2121.51 - 2500 + 3250 = 707.17x$

$$x = \frac{28.71.51}{707.17} = 4.06 \text{ m Ans.}$$

(*b*) Blocks *A* and *B*, f weight 150 N and 200 N, respectively rest on an inclined plane as shown in the figure. The coefficient of friction between the two blocks is 0.3 and between blocks *A* and inclined plane is 0.4. Find the value of θ for which either one or both the blocks start slipping. At that instant, what is the friction force between *B* and *A*? Between A and inclined plane?



Ans.



From the fig.

$R_1 = 200 \cos\theta$

$F_1 = 200 \sin\theta$

$F_1 = \mu R_1$

$200 \sin\theta = \mu\, 200 \cos\theta$

$\tan\theta = 0.3$

$\theta = 16.7°$ **Ans**

$F_1 = 200 \sin 16.7$

$F_1 = 57.47$ N **Ans.**

$R_2 - R_1 = 150 \cos\theta$

$F_2 - F_1 = 150 \sin\theta$

$F_2 = 350 \sin\theta$

$R_2 = 350 \cos\theta$

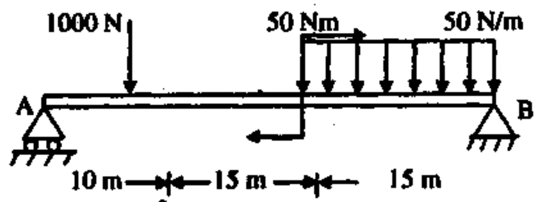
$F_2 = 350 \sin 16.7$

$F_2 = 100.6$ N

Body *B* will slide at $\theta = 16.7°$ **Ans.**

4. Attempt any one part of the following:

(*a*) For the beam shown in figure, draw the shear force and bending moment diagrams.



Ans. $\Sigma M_A = 0$

$$1000 \times 10 + 50 + 750 \times 32.5 = 40 \times R_B$$

$$10000 + 50 + 24375 = 40\ R_B$$

$$R_B = \frac{34425}{40} = 860.63\ N$$

$$R_A + R_B = 1000 + 50 \times 15$$

$$R_A + R_B = 1750\ N \quad ...(1)$$

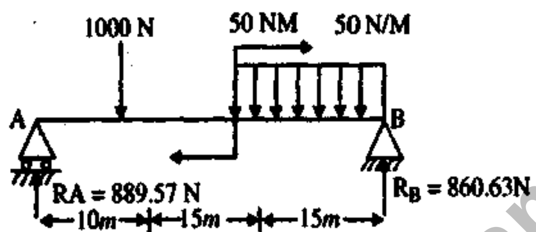


Fig (a)

Put the value of R_B in equation (1) we get the value of R_A

$$R_A = 1750 - 860.63$$
$$= 889.37\ N$$

For shear force diagram

$$SF_A = 889.37 \quad 0 \le x \le 10$$

at distance 25 *m*

$$SF = 889.37 - 1000 = -110.63 \quad 10 \le x \le 25$$
$$= -110.63 - 50\ (x - 25) \quad 25 \le x \le 40$$

→ at distance 25*m*

$$= 110.63 - 50\ (25 - 25)$$
$$= -110.63$$

→ at distance 40*m*

$$= -110.63 - 50\ (40 - 25)$$
$$= -110.63 - 50 \times 15$$
$$= -860.16\ kN.$$

with the help of *SF* draw the *SFD* show in fig (*b*)

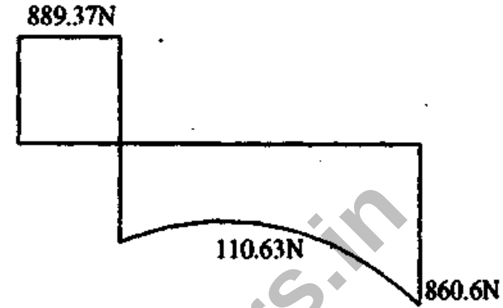


Fig (b)

For BMD: $M = 889.37\,x \quad 0 \le x \le 10$

at distance 0m $\quad M = 0$

at a distance 10 *m* = 8893.7

$$= 889.3x - 1000\ (x - 10) \quad 10 \le x \le 40$$

at distance 25 *m*

$$= 889.37 \times 25 - 1000\ (25 - 10)$$
$$= 22234.25 - 15000$$
$$= 72284.5\ kNM$$

$$= 889.37x - 1000\ (x - 10) + 50 - 25\ (x - 25)^2 \quad 25 \le x \le 40$$

at distance 25 *m*

$$= 889.3\,x - 1000\ (25 - 10) + 50 - 0$$
$$= -7234.25\ kNM$$

at distance 40 BM will be zero with the help of BM draw the BMD which is shown in fig (*c*)

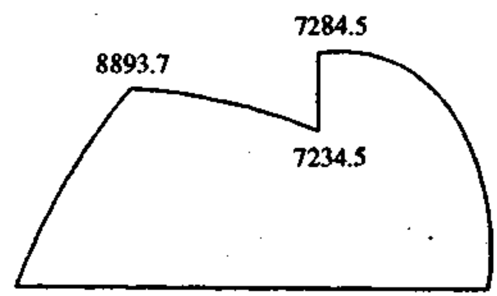
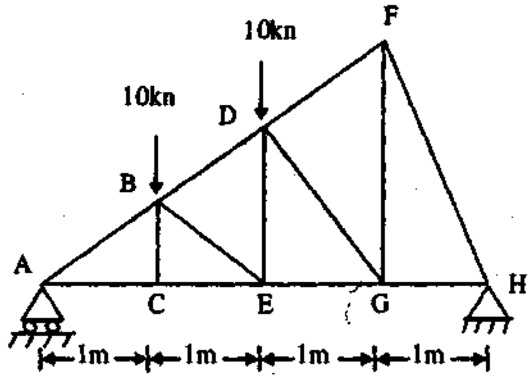
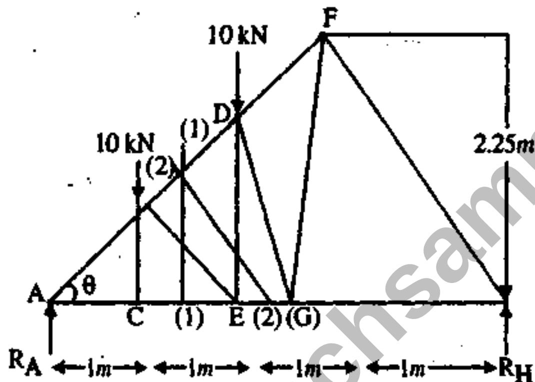


Fig (c)

(b) **For the simply supported transform the forces in the members *BD*.**



Ans. We calculated the force at members *BD*, DE, *EG* and *CE*



From Fig.

$$\tan\theta = \frac{2.25}{3} = 36.87°$$

$$R_A + R_H = 10 + 10 = 20 \text{ kN}$$

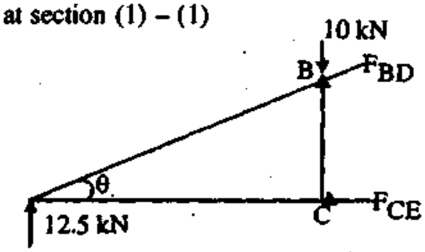
$$\Sigma M_A = 0$$

$$10 \times 1 + 10 \times 2 - R_H \times 4 = 0$$

$$R_H = 7.5 \text{ kN}$$

$$R_A = 12.5 \text{ kN}$$

at section (1) – (1)



$$\tan\theta = \frac{BC}{1}$$

$$\frac{2.25}{3} = \frac{BC}{1} \quad BC = 0.75$$

$$\Sigma M_B = 0$$

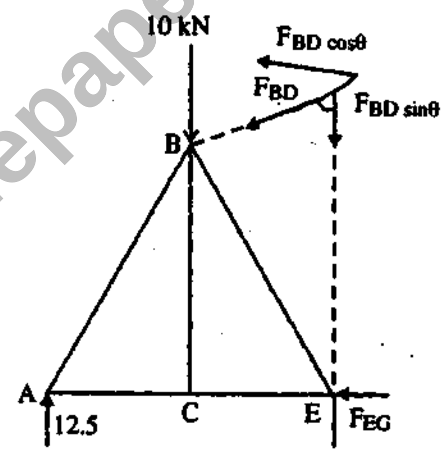
$$12.5 \times 1 + F_{CE} \times BC = 0$$

$$F_{CE} = \frac{-12.5}{BC} = \frac{-125}{0.75}$$

$$F_{CE} = 16.67 \text{ N } (T) \textbf{ Ans.}$$

at section (2) – (2)

From the fig.



$$\frac{1.25}{3} = \frac{DE}{2}$$

$$DE = 1.5 \text{ m}$$

$$\Sigma M_E = 0$$

$$12.5 \times 2 - 10 \times 1 - F_{BD} \cos 36.87 \times DE = 0$$

$$F_{BD} = \frac{25 - 10}{1.5 \times \cos 36.87} = 12.5 \text{ kN}(c)$$

$$F_{BD} = 12.5 \text{ kN}(c) \textbf{ Ans.}$$

$$\Sigma M_A = 0$$

$$10 \times 1 + F_{DE} \times 2 = 0$$

$$F_{DE} = 5 \text{ kN } (T) \textbf{ Ans.}$$

$$\Sigma M_B = 0$$

$$12.5 \times 1 + F_{DE} \times 1 + F_{EG} \times BC = 0$$

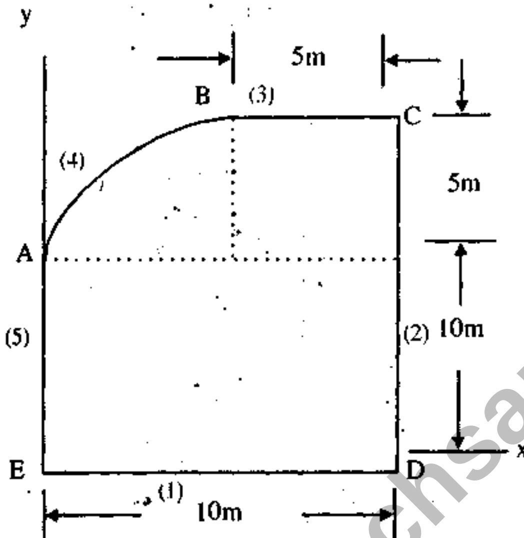
$$F_{EG} \times 0.7 = -12.5 + 5$$

$$F_{EG} = \frac{-7.5}{0.75} = 10 \text{ kN}$$

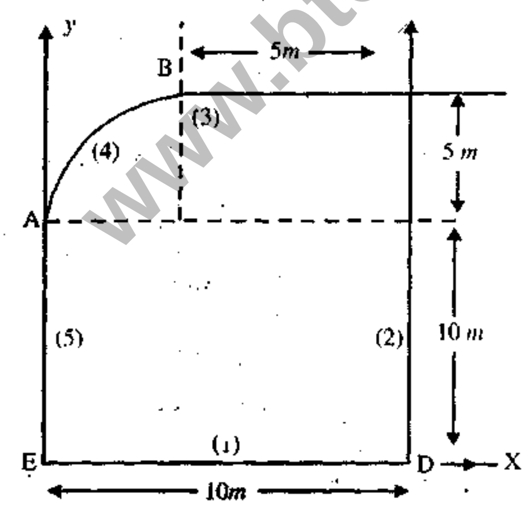
$$F_{EG} = 10 \text{ KN } (T) \text{ Ans.}$$

5. Attempt any two parts of the following:

(a) A wire is bent into a closed loop A-B-C-D-E-A as shown in figure in which portion AB is circular are. Determine the centroid of the wire.



Ans. From the given fig.



$l_1 = 10$ m, $l_2 = 15$ m

$l_3 = 5$ m

$$l_4 = \pi \times \frac{5}{2} = 7.85 \text{ m}$$

$l_5 = 10$ m

$\Sigma l = 10 + 15 + 5 + 7.85 + 10$

$= 47.85$ m

$x_1 = 5$ m, $x_2 = 10$ m

$$x_3 = 7.5 \text{ m } x_4 = 5 \times \frac{10}{\pi}$$

$= 1.817$

$x_5 = 0$

$\Sigma x = 24.317$

$y_1 = 0$, $y_2 = 7.5$m $y_3 = 15$ m

$$y_4 = 10 + \frac{10}{\pi} = 13.18 \text{ m}$$

$y_5 = 5$m

for the centriod of the wire

$$\bar{x} = \frac{\Sigma lx}{\Sigma l} = \frac{47.85 \times 24.317}{4}$$

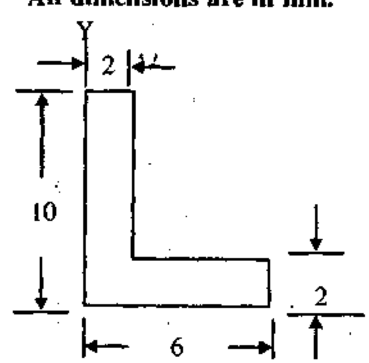
$= 60.8$ m

$$\bar{y} = \frac{\Sigma ly}{l} = \frac{47.85 \times 40.68}{4}$$

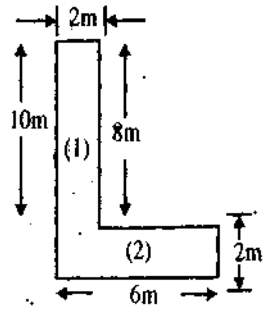
$= 7.1256$ m

Centriod of the wire will be at point (6.08, 7.1256) m.

(b) Find the principal moment of inertia about the origin of the are shown if figure.
All dimensions are in mm.



Ans. From the fig all dimensions in mm.



$a_1 = 8 \times 2 = 16\ mm^2$

$a^2 = 6 \times 2 = 12\ mm^2$

$x_1 = 1$ mm, $y_1 = 6$ mm

$x_2 = \frac{6}{2} = 3$ mm $y_2 = \frac{2}{2} = 1$ mm

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$= \frac{16 \times 1 + 12 \times 3}{16 + 12}$$

$$= \frac{52}{28} = 1.86 \text{ mm}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{16 \times 6 + 12 \times 1}{28}$$

$$= 3.86 \text{ mm}$$

$$I_{xx} = \frac{6 \times 2^3}{12} + 12\,(3.86 - 1)^2 + \frac{2 \times 8^3}{12} + 16 \times (6 - 3.86)^2$$

$$= \frac{48}{12} + 98.155 + 85.33 + 7327$$

$$= 260.72 \text{ mm}^4$$

$$I_{yy} = \frac{8 \times 2^3}{12} + (0.86)^2 \times 16 + \frac{2 \times 6^3}{12} + 12\,(3 - 1.86)^2$$

$$= 5.33 + 11.8336 + 36 + 15.56$$

$$= 68.76 \text{ mm}^4$$

$$I_{xy} = (8 \times 2) \times (-0.86) \times (6.3.86) + 12 \times (3 - 1.86) \times (3.86 - 2)$$

$$= -54.89 \text{ mm}^4$$

$$I_{1,2} = \frac{I_{xx} + I_{yy}}{2} \pm \frac{1}{2}\sqrt{(I_{xx} - I_{yy})^2 + 4(I_{xy})^2}$$

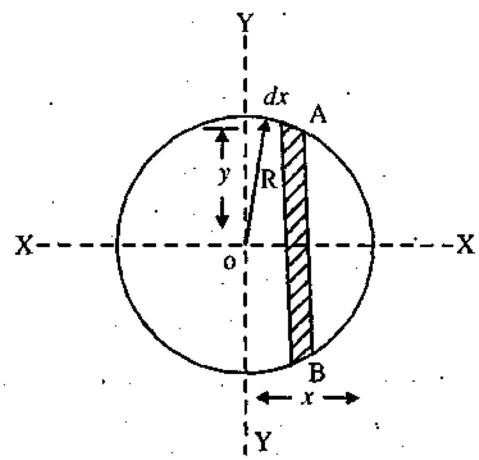
$$= \frac{260.72 + 68.76}{2} \pm \frac{1}{2}\sqrt{(260.72 - 68.76)^2 + 4 \times (-54.89)^2}$$

$$= 164.74 \pm 110.57$$

$I_1 = 275.31$ mm^4
$I_2 = 54.17$ mm^4 } **Ans.**

(c) Derive an expression for moment of inertia of a solid sphere about its diameter.

Ans. Figure shows a solid sphere of radius R with O as centre. If ρ is the density of the material of the sphere then,



Mass of sphere = density × volume

$$= \rho \times \frac{4}{3}\pi R^3$$

Let attention be focussed on a thin disc AB of thickness dx and at radius x from the centre.

radius of the disc $y = \sqrt{R^2 - x^2}$

Mass of the disc $dm = \rho \times \pi y^2 dx$

$= \rho\pi (R^2 - x^2)\, dx$

Mass moment of inertia of this elementary disc about the polar axis ZZ

$= dmy^2 = \rho\pi (R^2 - x^2)\, dx \times (R^2 - x^2)$

$= \rho\pi (R^2 - x^2)^2\, dx \times (R^2 - x^2)$

$= \rho\pi (R^2 - x^2)^2\, dx$

$= \rho\pi (R^4 + x^4 - 2R^2x^2)\, dx$

The mass moment of inertia of the whole sphere can be worked out by intergrating the above expression between the limits $-R$ to R

Mass moment of inertia of the sphere about polar axis ZZ

$$I_{ZZ} = \rho\pi \int_{-R}^{R} (R^4 - x^4 - 2R^2x^2)dx$$

$$I_{ZZ} = \rho\pi \left[R^4 x + \frac{x^5}{5} - 2R^2 \frac{x^3}{3} \right]_{-R_2}^{R}$$

$$= \frac{16\rho\pi R^5}{15} = \frac{4}{5} MR^2$$

Where $M = \frac{4}{3}\rho\pi R^3$ is the mass of the solid sphere, invoking the theorem of perpendicular axis, the mass moment of inertia of a solid sphere about xx or yy axis is

$$I_{xx} = I_{yy} = \frac{I_{22}}{2} = \frac{2}{5} MR^2 \text{ Proved}$$

6. Attempt any two parts of the following:

(a) The motion of a particle is defined by the relation $x = 6t^4 - 2t^3 - 3t + 3$. Determine the time, position, velocity and distnnce traveled when acceleration is zero.

Ans. Given relation

$$x = 6t^4 - 2t^3 - 12t^2 - 3t + 3$$

For velocity we know that

$$\frac{dv}{dt} = 0$$

$V = 24t^3 - 6t^2 - 24t - 3$ ... (1)

$a = 72t^2 - 12t - 24 = 0$ ... (2)

$a = 0 \Rightarrow 72t^2 - 12t - 24 = 0$

$V = 0\ 24t^3 - 6t^2 - 24t - 3 = 0$

Solve equation $72t^2 - 12t - 24 = 0$ for time

$$t = -b \pm \sqrt{\frac{b^2 - 4ac}{2a}}$$

$$t = +12 \pm \sqrt{\frac{144 - 4\times72\times(-24)}{2\times72}}$$

$$t = +12 \pm \sqrt{\frac{144 + 6912}{144}}$$

$$t = \frac{12 + \pm 84}{144}$$

For

$$t = \frac{12 + 84}{144} = 0.67 \text{ Sec } \textbf{Ans.}$$

t will be 0.67 second because negative value not valid for the time put the value of t in equation (1) get the value of V

$V = 24(0.67)^3 - 6(0.67)^2 - 24 \times 0.67 - 3$

$V = 7.22 - 2.69 - 16.77 - 3$

$V = -14.55$ m/sec **Ans.**

put the value of t we get the value of x.

$x = 6t^4 - 2t^3 - 12t^2 - 3t + 3$

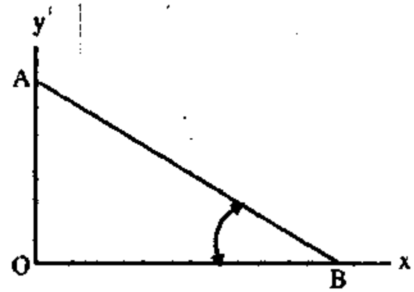
$= 6(0.67)^4 - 2(0.67)^3 - 12 \times (0.67)) + 3$

$x = -3.74$ m **Ans.**

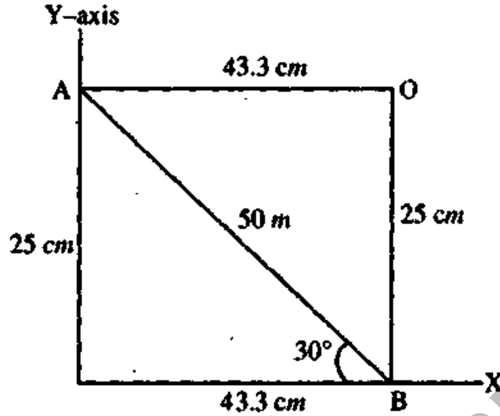
distance travelled $= 3 + 3.74$

$= 6.74$ m **Ans.**

(b) A straight rigid link AB of length 50 cm is shown is the figure. The end B of the link moves along x-axis with a velocity of 4 m/s and accelerates with an acceleration of 10 m/s^2. The end A is constrained to move along y-axis. Find the velocity and acceleration of the end A at the given instant.



Ans. Given data



$AB = 50$ m

$V_B = 4$ m/sec

$a_B = 10$ m/sec2

$V_B = \omega \times r = \frac{4}{0.25}$

$\omega = 16$ rad/sec

$V_A = \omega \times OA$

$= 16 \times 0.433$

$= 6.928$ m/sec **Ans.**

From the fig.

$$\sin 30 = \frac{OA}{50}$$

$OA = 50 \times \sin 30$

$$= 50 \times \frac{1}{2} = 25 \text{ cm}$$

$a_B = \alpha \times r$

$$\alpha = \frac{a_B}{r} = \frac{10}{0.25} = 40 \text{ rad/sec}^2$$

$a_A = \alpha \times OA$

$= 40 \times 0.433$

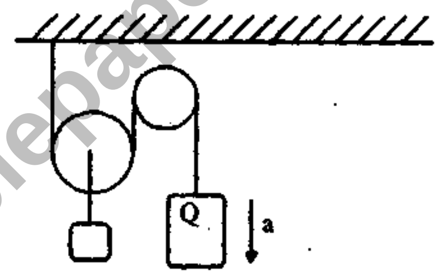
$= 17.32$ m/sec2 **Ans.**

$$OB = \sqrt{(50)^2 - (25)^2}$$

$$= \sqrt{250 - 625}$$

$= 43.3$ cm

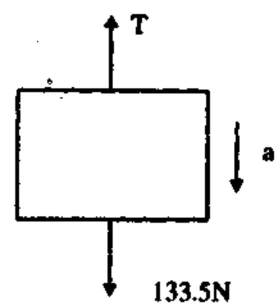
(*c*) Two weight *P* and *Q* are connected by the arrangement shown in figure. Neglecting friction and the inertia of the pulleys and cord, find the acceleration a of the weight *Q*. Assume that *P* = 178 N and *Q* = 133.5 N.



Ans. Given $P = 178$ N, $Q = 133.5$ N

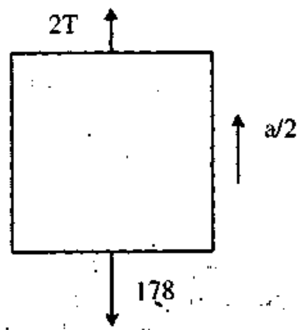
For the equilibrium condition

$$133.5 - T = \frac{133.5}{g} a \quad \text{...(1)}$$



$$2T - 178 = \frac{178}{g} \times \frac{a}{2} \quad \text{...(2)}$$

equation (1) × 2 + equation (*ii*)



Free body diagram

$$267 - 2T = \frac{267}{g}.a$$

$$2T - 178 = \frac{178}{g}.\frac{a}{2}$$

$$89 = \frac{267}{g}.a + \frac{178}{g}.\frac{a}{2}$$

$$89 = \frac{a}{g}[267 + 89]$$

$$89 = \frac{a}{g}[356]$$

$$a = \frac{89 \times g}{356} = \frac{89 \times 98.1}{356}$$

$a = 2.454$ m/sec^2 **Ans.**

7. Attempt any two parts of the following:

(*a*) Define Poisson's Prove that its value lies between zero and half.

Ans. Poisson's Ratio: It has been experimentally found, that if a body is stressed within elastic limit, the lateral of strain bears a constant ratio to the linear strain. Mathematically

$$\frac{\text{Leteral strain}}{\text{Linear strain}} = A \text{ (constant)}$$

Thus constant as known as poisons ratio and is dentoed by $\frac{1}{M}$ or μ. Thus we see that

$$\text{Later strain} = \frac{1}{M} e = \mu.e$$

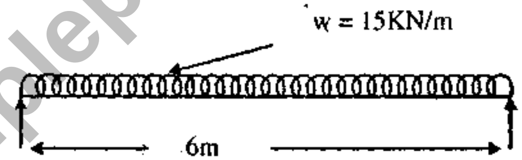
The corresponding change in the lateral length may be found out, as usual, *i.e.*, by multiplying the lateral strain by the lateral length (*i.e.* width or thickness).

(*b*) Determine the dimensions of a simply supported reetangular steel beam 6 m long to carry a brick wall 250 mm thick and 3 m high, if the brick weighs 20 kN/m^3 and maximum permissible bending stress is 800 N/cm^2. The depth of a beam is 1.50 times its width.

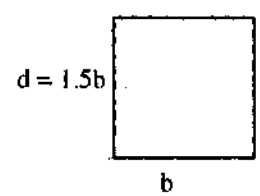
Ans. Given $l = 6$ m, $d = 1.5\ b$

$$W = 20 \times 0.25 \times 3 = 15 \text{ kN/m}$$

$$\sigma_b = 800 \text{ N/cm}^2$$



we know the in case of simply supported beam the max B.M is



$$M_{max} = \frac{Wl^2}{8} = \frac{15 \times 6 \times 6}{8}$$

$$= 67.5 \text{ kN.m}$$

$$I = \frac{bd^3}{12} = \frac{b(1.5b)^3}{2}$$

$$= 0.28125\ b^4$$

$$y_{max} = \frac{d}{2} = \frac{1.5b}{2} = 0.75\ b$$

$$\sigma_b = \frac{M_{max}\ y_{max}}{I}$$

$$8 \times 10^6 = \frac{67.5 \times 10^3 \times 0.75\,b}{0.28125\,b^4}$$

$$8 \times 10^6 \times 0.28125\,b^4 = 50625\,b$$

$$2250000\,b^3 = 50625$$

$$b^3 = 0.0225$$

$$b = 0.2823 \text{ m}$$

$$b = 28.23 \text{ cm Ans.}$$

$$d = 1.5 \times 28.23$$

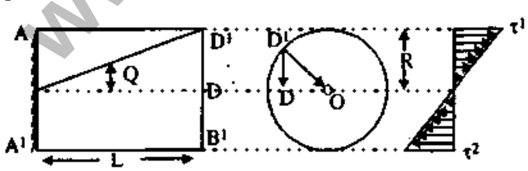
$$d = 42.346 \text{ cm Ans.}$$

(c) For torsion of a circular shaft, derive the torsion equation. State all the assumptions at the beginning.

Fig.

Ans. Assumptions of deriving the Torsional formulas: The torsion equation based on the following assumptions.

1. The material of the shaft is uniform throughout
2. The shaft circular in cross-section remains circular after loading .
3. A plane section of shaft normal to its axis before loading remains plane after the torques has been applied.
4. The twist along the length of shaft is uniform throughout
5. The distance between any two normal cross-chains remains the same after the application of toque.
6. Maximum shear stress induced in the shaft due to application of torque does not exceed its elastic limit value.



Torsion equation,

$$\frac{T}{J} = \frac{\tau}{R} = \frac{GQ}{L}$$

Let T = maximum twisting torque or twisting moment

D = Diameter of shaft

R = Radius of the shaft

τ = Max. permissible shear stress

G = Modulus of Rigidity

θ = Angle of twist (radians) = angle $D'OD$

L = Length of the shaft

ϕ = Angle $D'CD$ = Angle of shear strain

Let the shaft is subjected to a torque or twisting moment 'T' and hence every C.S. of this shaft will be subjected to shear stress.

Now distortion at the outer surface = DD'

$$\text{Shear strain at outer surface} = \frac{\text{Distortion}}{\text{Unit length}}$$

i.e. Shear stress at the outer surface (f is very small) .

$$\left(\tan\phi + \frac{DD'}{CD} = \frac{DD'}{L}\right)$$

$$\text{or } \phi = \frac{DD'}{L} = \phi = \frac{R.\theta}{L} \quad [\text{since } DD' = R.\theta]$$

$$G = \frac{\text{Shear stress induced}}{\text{Shear strain produced}}$$

$$G = \frac{\tau}{\left(\frac{R\theta}{L}\right)} \text{ or } \boxed{\frac{\tau}{R} = \frac{G\theta}{L}}$$

This equation called stiffness equation.

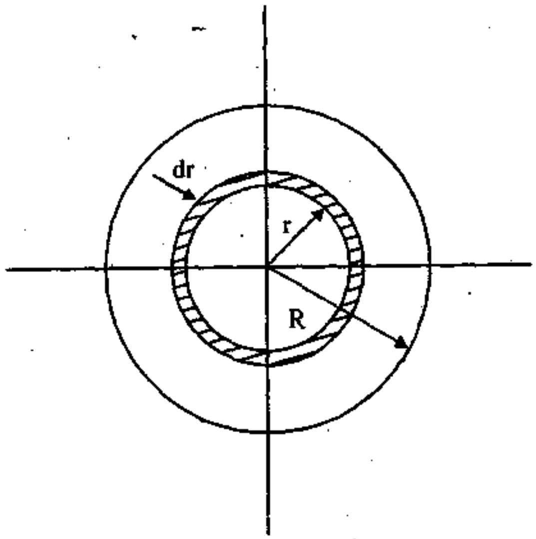
Hear G, θ, L are constant for a given torque 'T'

i.e. τ is proportional to R.

Now torque is terms of polar moment of inertia.

$$\text{From fig.2 } \tau_R = \left(\frac{\tau}{R}\right).r$$

Turning force an elementary Ring



$$\left(\frac{\tau}{R}\right).r.\,dA$$

Turning moment

$$dT = \left(\frac{\tau}{R}\right).r.r\,dA$$

$$dT = \left(\frac{\tau}{R}\right) r^2\,dA$$

$$T = \left(\frac{\tau}{R}\right) \int_0^R r^2\,dA \qquad \text{... (2)}$$

$\int_0^R r^2\,dA$ = M.I of elementry ring = J (polr M.I.)

Now for equation (2)

$$T\left(\frac{\tau}{R}\right).J \text{ or } \boxed{\frac{\tau}{R} = \frac{T}{J}} \qquad \text{... (3)}$$

Thus equation is called strength equation combined equation (1) and (3) we get

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G.\theta}{L} \text{ Proved}$$

This equation is called torsion equation.