

FIRST SEMESTER EXAMINATION, 2007-2008

PHYSICS

Time : 3 Hours

Total Marks : 100

Note. (1) Attempt all questions.

(2) Marks of each question are shown against it.

(3) The physical constants are given at the end of the question paper.

Q.1. Answer any four of the following:

(5×4=20)

1. (a) What do you understand by a frame of reference? Is earth an inertial frame of reference? If not, why?

Ans. In order to specify the location of a point object or an event in space, a coordinate system is required. Such a coordinate system with respect to which we measure the position of a point object or an event is called a frame of reference.

Earth can not be treated as an inertial frame of reference, because of its rotational motion about its axis. This rotation motion evolves a fictitious force on the object.

1. (b) Calculate the percentage contraction in the length of rod in a frame of reference, moving with velocity $0.8c$ in a direction parallel to its length.

Ans. Length of the moving rod

$$\begin{aligned} L &= L_0 \sqrt{1 - v^2/c^2} \\ &= L_0 \sqrt{1 - (0.8c)^2/c^2} \\ &= L_0 \sqrt{1 - 0.64} \\ &= L_0 \sqrt{0.36} = L_0 \times (0.6) \end{aligned}$$

where L_0 is the proper length

$$\begin{aligned} \therefore \% \text{ contraction} &= \frac{L_0 - L}{L_0} \times 100 \\ &= \frac{L_0 - 0.6L_0}{L_0} \times 100 \\ &= \frac{0.4L_0}{L_0} \times 100 = 40\%. \end{aligned}$$

1. (c) Obtain the relativistic formula for addition of velocities and also show that the speed of light is constant.

Ans. We know from the relation between relativistic energy and momentum.

$$E^2 = p^2 c^2 + m_0^2 c^4 \quad \dots(1)$$

where E is total energyi.e. $E = K_E + \text{rest mass energy}$

$$= KE + m_0 c^2$$

Substituting the value of E from equation (2) to equation (1) we get,

$$\begin{aligned} (K_E + m_0 c^2)^2 &= p^2 c^2 + m_0^2 c^4 \\ KE^2 + m_0^2 c^2 + 2K_E m_0 c^2 &= p^2 c^2 + m_0^2 c^4 \\ \Rightarrow p^2 c^2 &= K_E^2 + 2K_E m_0 c^2 \\ \Rightarrow p^2 &= \frac{K_E^2}{c^2} + 2K_E m_0 \end{aligned}$$

$$\Rightarrow p = \sqrt{\frac{K_E^2}{c^2} + 2K_E m_0} \quad \text{Proved.}$$

1. (d) A circular lamina moves with its plane parallel to the $x - y$ plane of a reference frame S , at rest. Assuming its motion to be along the axis of x (or y), calculate the velocity at which its surface area would appear to be reduced to half to an observer in frame S' .

Ans. The circular lamina moving along x or y axis will appear to be an ellipse for an observer in stationary frame of reference S . Let D_c be the diameter of the circle. Therefore

$$D_x = D_c \sqrt{1 - \frac{v^2}{c^2}} \quad \dots(1)$$

But we can write the area of circular lamina

$$= \pi \left(\frac{D_c}{2} \right)^2 \quad (\because \pi r^2 \text{ is the area}).$$

Similarly area of elliptical lamina = $\pi.ab$

$$\begin{aligned} &= \pi \left(\frac{D_c}{2} \right) \left(\frac{D_c}{2} \sqrt{1 - \frac{v^2}{c^2}} \right) \\ &= \pi \frac{D_c^2}{4} \sqrt{1 - \frac{v^2}{c^2}} \quad \dots(2) \end{aligned}$$

According to the question

$$\pi \frac{D_c^2}{4} \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2} \pi \left(\frac{D_c}{2} \right)^2 = \frac{1}{2} \pi \frac{D_c^2}{4}$$

$$\text{So, } \frac{\pi D_c^2}{4} \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2} \pi \frac{D_c^2}{4}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2}$$

$$\text{or } 1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$\text{or } \frac{v}{c} = \frac{\sqrt{3}}{2}$$

$$\text{or } v = \frac{\sqrt{3}}{2} \times c$$

$$= \frac{\sqrt{3}}{2} \times 3 \times 10^8$$

$$= 1.732 \times 1.5 \times 10^8 \text{ m/s}$$

$$\Rightarrow 2.598 \times 10^8 \text{ m/s}$$

1. (e) What was the objective of Michelson-Morley experiment? Discuss the negative result of this experiment.

Ans. Michelson Morley Experiment: Michelson Morley performed a historic optical experiment to check if the velocity of light measured with respect to earth moving through ether actually depended upon the velocity of the earth.

Experimental arrangement (Fig. 1) for the Michelson Morley experiment is shown below Fig. 1. S is any monochromatic source, a parallel beam falls upon a glass plate P , which is inclined 45° from the horizontal and plate is semisilvered polish. About half the light gets reflected at the back surface along mirror M_1 , whereas other half gets refracted at the both surfaces to emerge parallel to the original beam along M_2 . It also consists of plate Q such that P and Q are of equal thickness and of the same material, mounted parallel to each other.

The reflected beam moves towards mirror M_1 and at A it falls normally, hence it is reflected back to P and other being transmitted from P enters the telescope T . Similarly transmitted beam moves towards mirror M_2 and falls normally at B after entering the plate Q . The beam reflected at P crosses the plate P twice while other beam in the absence of Q lies wholly in air. So the reflected beam at P has to travel an extra optical path $2(\mu - 1)t$, μ is refractive index of the P and t is its thickness. The function of the plate Q is only to equalise the optical paths traversed by the both the beams if mirrors M_1 and M_2 are at equal distances from P .

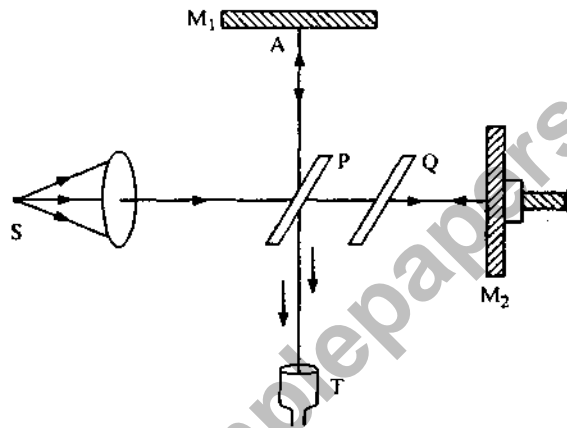


Fig. 1

Let us suppose that the two mirrors be at equal distances from the plate P , i.e. $PA = PB = l$ (say).

We have supposed that velocity of the apparatus is V . So the ray coming from P strikes the mirror M_1 at A' instead of A due to the motion of the earth. (Fig. 2)

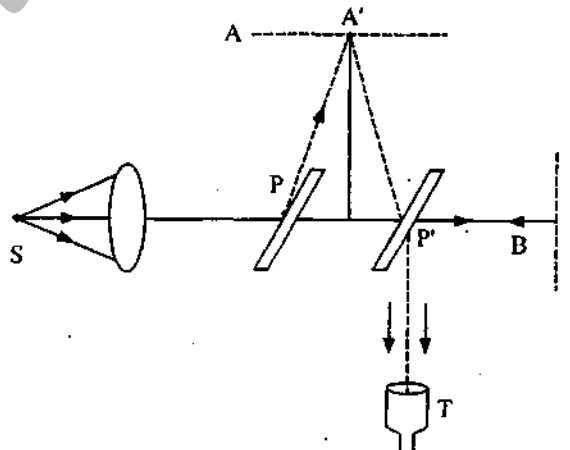


Fig. 2

Hence, total path travelled by reflected beam = $PA'P' = PA' + A'P' = 2PA'$ (since $PA' = PA'$)

From the triangle $PA'O$, we have,

$$(PA')^2 = (PO)^2 + (OA')^2 \quad \dots(1)$$

Again supposing that time taken by the beam to reach from P to mirror M_1 , then

$$PA' = ct \text{ and } AA' = vt \text{ and } OA' = l$$

Therefore equation (1) can yields the form

$$c^2t^2 = v^2t^2 + l^2$$

$$\text{or } t = \frac{l}{\sqrt{c^2 - v^2}}$$

If t_1 time is taken by the reflected beam to complete total distance, then

$$t_1 = 2t = \frac{2l}{\sqrt{c^2 - v^2}}$$

$$= \frac{2l}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{2l}{c} \left\{ 1 - \frac{v^2}{c^2} \right\}^{-1/2}$$

$$= \frac{2l}{c} \left(1 + \frac{v^2}{c^2} \right)$$

(By binomial theorem) $\dots(2)$

The transmitted beam has velocity $(c - v)$ because the direction of travel of light is along the direction of motion of apparatus, similarly $(c + v)$ along opposite direction of the light.

So total time taken by transmitted beam to travel the total path i.e., from P to B' and from B' to P' is t_2 ,

Therefore

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v} \text{ (since } PB = P'B' = l)$$

$$= \frac{2l}{c^2 - v^2}$$

$$= \frac{2l}{c^2 \left(1 - \frac{v^2}{c^2} \right)}$$

$$= \frac{2l}{c^2} \left(1 - \frac{v^2}{c^2} \right)^{-1} = \frac{2l}{c} \left(1 + \frac{v^2}{c^2} \right)$$

$\dots(3)$

Therefore, the time difference

$$\Delta t = t_2 - t_1$$

$$= \frac{2l}{c} \left\{ 1 + \frac{v^2}{c^2} \right\} - \frac{2l}{c} \left\{ 1 + \frac{v^2}{2c^2} \right\}$$

$$= \frac{2l}{c} \frac{v^2}{2c^2} = \frac{l}{c} \frac{v^2}{c^2} = \frac{lv^2}{c^3} \quad \dots(4)$$

Due to this time difference, the waves which are in phase at the time separate at P will not be in phase on return. The time difference should appear as fringe shift from the position the fringe would occupy, if there were no other drift. So the difference of path will be in opposite direction, the total shift of fringe is equivalent to $\frac{2l v^2}{c^2 \lambda}$.

Negative results of Michelson Morley experiment:

The experiments were repeated at various locations and at different times of the year, but always yielded null results. The following interpretations were offered to explain the null results of the experiment.

1. Ether-Drift Hypothesis: The earth might drag a little ether along with it near its surface. In another words, the moving earth drags the ether along with it and hence there is no relative motion between the earth and ether.

2. Lorentz-Fitzgerald Contraction Hypothesis: A brilliant suggestion to explain the negative results of Michelson Morley experiment, was given by Fitzgerald and Lorentz. They proposed that the dimensions of all material bodies get contracted by a factor

$\sqrt{1 - \frac{v^2}{c^2}}$ in a direction parallel to the relative velocity. In Michelson Morley experiment, the Fitzgerald contraction means that when the apparatus is in motion, the distance travelled by the beam B is not l , but $l\sqrt{1 - \frac{v^2}{c^2}}$. Thus

Fitzgerald contraction theory is able to explain the null results of Michelson Morley experiment, it is purely hypothetical.

3. Constancy of Speed of Light: As Einstein proposed that light's speed is invariant, means speed of light in never depends or changes upon the motion of the source.

The essential conclusion, which follows from the results of Michelson Morley experiment is that it is not possible to measure the relative velocity of the earth relative to ether.

1. (f) Show, from Lorentz transformation that two events simultaneous ($t_1 = t_2$) at different positions ($x_1 \neq x_2$) in a reference frame s are not, in general, simultaneous in another reference s' moving with constant velocity v w.r.t. s .

Ans. Suppose (x, y, z, t) and (x', y', z', t') are the coordinates of the same event observed by the observers in stationary frame of reference S and in moving frame of reference S' respectively. Also let us consider that the moving frame of reference S' is moving with velocity v along $+ve$ X direction.

We have to prove using Lorentz transformation that

$$x'^2 + y'^2 + z'^2 + c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2 \quad \dots(1)$$

The Lorentz transformation equations are

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, y' = y, z' = z,$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Substituting the values of x', y', z' and t' in LHS to equation (1) we get

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = \frac{(x - vt)^2}{\left(1 - \frac{v^2}{c^2}\right)} + y^2 + z^2 - c^2 \frac{\left(t - \frac{vx}{c^2}\right)^2}{1 - v^2/c^2}$$

According to the Lorentz-Fitzgerald length contraction the coordinates in S' frame which is moving with velocity v along x -axis is

$$x' = x\sqrt{1 - \frac{v^2}{c^2}}$$

$$y' = y$$

$$\Rightarrow x = \frac{x'}{\sqrt{1 - v^2/c^2}} \text{ and } y = y'$$

Substituting the values of x and y in the eqn. of circle we get,

$$\frac{x'^2}{\sqrt{1 - v^2/c^2}} + y'^2 = a^2$$

$$\Rightarrow \frac{x'^2}{a^2\left(1 - \frac{v^2}{c^2}\right)} + \frac{y'^2}{a^2} = 1$$

$$\Rightarrow \frac{x'^2}{b^2} + \frac{y'^2}{a^2} = 1$$

$$\text{Where } b^2 = a^2\left(1 - \frac{v^2}{c^2}\right)$$

Thus in the moving frame S' the circle will appear as an ellipse.

2. Attempt any four of the following: (5×4=20)

2. (a) What are the coherent sources of light? Is it possible to obtain coherent

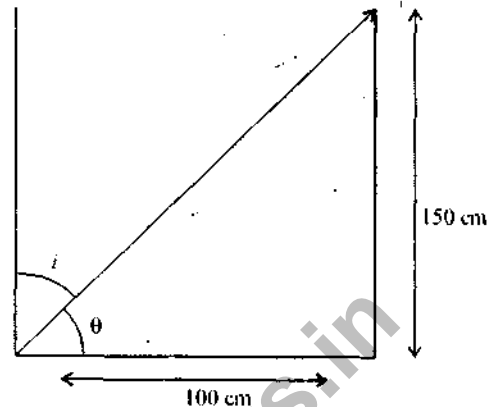
sources from two separate sources?
If not, why?

Ans. Coherent Sources: Two sources of light are said to be coherent if they emit light which have always a constant phase difference between them. It means that the two sources must emit radiations of the same wavelength. The two independent sources cannot be coherent because of the fact that independent sources cannot maintain a constant phase difference between them. For experimental purposes, two virtual sources obtained from a single parent source can act as coherent. In such case all the random phase change occurring in the parent source are repeated in the virtual sources also, thus maintaining a constant phase difference between them. Since the wavelength of light waves is extremely small, the two sources must be narrow and must also be close to each other.

No Interference by Two Independent Light Sources: Sustained interference can never be obtained with two independent sources of light, such as two bulbs or two candles. It is due to the fact that any two independent beams of light are always incoherent. Actually a beam of light is built up of waves radiated from millions of excited atoms or molecules, whose vibrations are completely independent of each other, so that the initial phase of the beam is absolutely governed by chance. After a time interval of 10^{-8} sec, this phase will be randomly changed because of the fact that the excited atoms which are responsible for vibrations are replaced by other excited atoms. Thus every source undergoes haphazard changes of phase in very billionth of a second. Hence the resultant intensity on the screen varies rapidly with time and two incoherent sources cannot produce any stationary interference pattern.

2. (b) A man whose eyes are 150 cm above the oil film on water surface observes greenish colour at a distance of 100 cm from his feet. Calculate the probable thickness of the film.

($\lambda_{\text{green}} = 5000 \text{ \AA}$, $\mu_{\text{oil}} = 1.4$, $\mu_{\text{water}} = 1.33$)
Ans.



$$\tan \theta = \frac{150}{100}$$

\therefore Angle of incidence

$$i = 90 - \theta$$

$$= 90 - \tan^{-1}(1.5) = 90 - 56.31$$

$$i = 33.69^\circ$$

For the greenish colour the thickness required by the formula.

$$2\mu t \cos r = (2n - 1)\lambda/2 \quad \text{for } n = 1, 2, 3, \dots$$

For minimum thickness

$$t = \frac{\lambda}{2 \times 2\mu \cos r}$$

$$\cos r = \sqrt{1 - \sin^2 r}$$

$$= \sqrt{1 - \left(\frac{1.33}{1.44} \sin i\right)^2}$$

$$= \sqrt{1 - 0.28} = 0.85$$

$$\text{We have } \mu = \frac{\sin i}{\sin r}$$

$$\frac{1.49}{1.33} = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{1.33}{1.40} \sin i = 0.53$$

$$\therefore t = \frac{5000}{2 \times 2 \times 1.4 \times 0.85}$$

$$l = 1050.4 \text{ \AA} \quad \text{Ans.}$$

2. (c) In Newton's ring experiment what happens when
 (i) a point source of light is used
 (ii) light is not monochromatic
 (iii) plane glass plate is replaced by plane mirror.

Ans. (i) If a point source is used to illuminate a thin film, the light reflected from all corresponding point on the film does not reach the eye. The rays only from a small portion of the film can enter the eye i.e. microscope. Hence the entire rings can not be seen by the microscope placed in a fixed position.

(ii) When an air film between the planes convex lens and glass plate is illuminated with a non monochromatic light, a few mixed coloured rings around a black center are observed and beyond it a uniform illumination is obtained. This is because the diameters of the ring is a function of wavelength and white light is composed of a number of colours (wavelengths).

(iii) If the top surface of the glass plate on which lens is kept is a mirror, the ring on the reflected system would disappear and a uniform illumination is observed. It is due to the fact that, there would be no transmission of the rays, but transmitted rays will also be reflected at the silvered surface and the two complementary systems of rings super impose on each other and gives a uniform illumination.

2. (d) A diffraction grating used at normal incidence given a green line (5400 Å) in a certain order superimposed on the violet line (4050 Å) of the next higher order. If the angle of diffraction is 30°, how many lines per cm are there in the grating?

Ans. We know from grating equation

$$(d + d) \sin \theta = n\lambda \quad \dots(i)$$

where $(e + d)$ is the grating element

n is the order

λ is the wavelength of light

θ is the angle of diffraction

According to question,

For yellow line

$$(e + d) \sin \theta = n \times 6000 \quad \dots(ii)$$

For blue line

$$(e + d) \sin \theta = (n + 1)48000 \quad \dots(iii)$$

From (ii) and (iii)

$$n \times 6000 = (n + 1)4800$$

$$\therefore n = 4$$

Substituting the value of n in equation (ii)

$$\text{We get } (e + d) \sin \theta = 4.6000$$

$$(e + d) \cdot \left(\frac{3}{4}\right) = 4 \times 6000$$

$$\therefore (e + d) = \frac{4 \times 6000 \times 4}{3} \text{ \AA} \\ = 3.2 \times 10^4 \text{ cm}$$

2. (e) What do you understand by missing order spectrum? What particular spectra would be absent if the width of transparencies and opacities of grating are equal?

Ans. We have

$$\frac{e + d}{e} = \frac{n}{m} \text{ or } n = \left(\frac{e + d}{e}\right)m$$

$$m = 1, 2, 3, \dots$$

According to the question

$$\text{if } e = d$$

$$\text{then } n = \frac{2d}{d}m$$

$$n = 2m \text{ for } m = 1, 2, 3, \dots$$

So 2nd, 4th, 6th, 8th etc.

Orders are absent.

2. (f) Two pin holes 1.5 mm apart are placed in front of a source of light of wavelength 5.5×10^{-5} cm and seen through a telescope with its objective stopped down to a diameter of 0.4 cm. Find the maximum distance from the telescope at which the pin holes can be resolved.

Ans. Distance between the holes $x = 1.5 \text{ mm}$

$$l = 5.5 \times 10^{-5} \text{ cm}$$

Diameter $a = 0.4$ cm

The smallest angular separation that can be resolved by a telescope objective of diameter a is given by

$$\theta = \frac{1.22\lambda}{a}$$
$$= \frac{1.22 \times 5.5 \times 10^{-5}}{0.5}$$

$$\theta = 1.60 \times 10^{-4}$$

$$\theta = \frac{x}{l}$$

where l is distance between holes and Telescope.

$$\text{So } l = \frac{x}{\theta}$$
$$= \frac{1.5 \text{ mm}}{1.68 \times 10^{-4}}$$
$$= 8.92 \times 10^{-3} \text{ mm}$$
$$= 8.92 \text{ m} \quad \text{Ans.}$$

3. Attempt any four of the following: (5×4=20)

3. (a) The values of μ_E and μ_O for quartz are 1.5508 and 1.5418 respectively. Calculate the phase retardation for $\lambda = 5000 \text{ \AA}$ when the plate thickness is 0.032 mm.

Ans. In this case the path difference between E-ray and O-ray

$$= (\mu_E - \mu_O)t$$

As, phase difference

$$= \frac{2\pi}{\lambda} (\text{path diff.})$$
$$= \frac{2\pi}{\lambda} (\mu_E - \mu_O)t$$
$$= \frac{2 \times 3.14(1.5508 - 1.5418) \times 0.0032}{5000 \times 10^{-8}}$$

$$= \frac{6.28 \times 0.0090 \times 0.0032}{5 \times 10^{-5}}$$

$$= 36.17 \times 10^{-6+5}$$

$$= 3.6 \text{ radians} \quad \text{Ans.}$$

3. (b) Show that the plane polarised and circularly polarised light are the special cases of elliptically polarised light.

Ans. Plane, Circularly and Elliptically Polarised Light:

Plane polarised light: When light travels along a certain direction the vibrations take place in a direction to right angles to the direction of propagation. If the vibrations of the ether particles are linear and take place parallel to a plane through the axis of the beam or the direction of propagation, light is said to be plane polarised.

Elliptically polarised light: When the vibrations of the ether particles are elliptical, having a constant period and take place in a plane perpendicular to the direction of propagation, light is said to be elliptically polarised. In elliptically polarised light the amplitude of the vibrations changes in magnitude as well as in direction.

Circularly polarised light: When the vibrations of the ether particles are circular having constant period and take place in the transverse plane, light is said to be circularly polarised. In circularly polarised light, the amplitude of the vibrations remains constant but the direction changes only.

Superposition of Two Linearly Polarised Sinusoidal Light Waves: Let a beam of linearly polarised light be incident normally on a calcite plate cut with its optic axis parallel to its faces. Let the linear vibration in the incident light be along PA (Fig.), making an angle θ with the optic axis. Let A be the amplitude of vibration in the incident light.

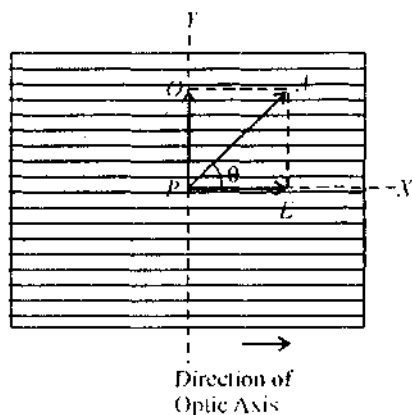


Fig. 1

On entering the crystal, the amplitude of the incident light wave splits up into two components, $A \cos \theta$ along PE (E -ray) and $A \sin \theta$ along PO (O -ray). The component $A \cos \theta$ having vibrations parallel to the direction of optic axis and $A \sin \theta$ having vibrations perpendicular to the optic axis. Now in calcites crystal, as $\mu_o > \mu_e$, then velocity of O -ray is less than E -ray (i.e. $V_E > V_O$). Hence E -ray emerges from the plate leading with a phase difference δ with O -ray.

If $A \sin \omega t$ is the incident wave, then the two emergent linearly polarised E - and O -waves would be represented by:

$$E\text{-ray: } x = A \cos \theta \sin (\omega t + \delta) \quad \dots(1)$$

$$O\text{-ray: } y = A \sin \theta \sin \omega t \quad \dots(2)$$

$$\text{Let } A \cos \theta = a \text{ and } A \sin \theta = b$$

$$\text{Then, } x = a \sin (\omega t + \delta) \quad \dots(3)$$

$$y = b \sin \omega t \quad \dots(4)$$

Now eqn. (3) can be written as

$$\begin{aligned} \frac{x}{a} &= \sin \omega t \cos \delta + \cos \omega t \sin \delta \\ &= \sin \omega t \cos \delta + \sqrt{1 - \sin^2 \omega t} \sin \delta \end{aligned}$$

From eq. (4)

$$\sin \omega t = \frac{y}{b}$$

$$\therefore \frac{x}{a} = \frac{y}{b} \cos \delta + \sqrt{1 - \frac{y^2}{b^2}} \sin \delta$$

$$\left(\frac{x}{a} - \frac{y}{b} \cos \delta \right)^2 = \left(1 - \frac{y^2}{b^2} \right) \sin^2 \delta$$

$$\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \delta + \frac{y^2}{b^2} \cos^2 \delta$$

$$= \sin^2 \delta - \frac{y^2}{b^2} \sin^2 \delta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} (\cos^2 \delta + \sin^2 \delta) - \frac{2xy}{ab} \cos \delta = \sin^2 \delta$$

$$[\because \cos^2 \delta + \sin^2 \delta = 1]$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \delta = \sin^2 \delta \quad \dots(5)$$

Eq. (5) in general represents an *ellipse*. Hence the light emerging from the crystal plate is, in general, elliptically polarised.

Now taking special cases:

(i) If the thickness of the plate be such that $\delta = 0, 2\pi, 4\pi, \dots$ then $\cos \delta = 1$ and $\sin \delta = 0$.

Then eq. (5) becomes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\text{or } \left(\frac{x}{a} - \frac{y}{b} \right)^2 = 0$$

$$\text{or } \pm \left(\frac{x}{a} - \frac{y}{b} \right) = 0$$

$$\text{or } \pm y = \pm \frac{b}{a} x \quad \dots(6)$$

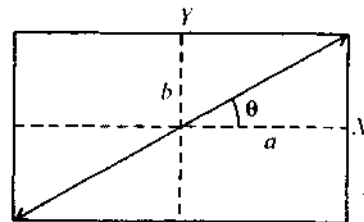


Fig. 2

This represents a pair of coincident straight

lines through origin have a positive slope $\frac{b}{a}$ (Fig.

2). This means that the emergent light is plane-polarised with the same direction of vibration as the incident light.

$$(ii) \quad \delta = \pi, 3\pi, 5\pi, \dots$$

then $\cos \delta = -1$ and $\sin \delta = 0$

Then eq. (15) becomes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

$$\text{or } \pm \left(\frac{x}{a} + \frac{y}{b} \right) = 0$$

$$\text{or } \pm y = \mp \frac{b}{a} x \quad \dots(7)$$

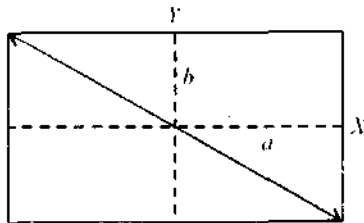


Fig. 3

Again the represents a plane-polarised light with the vibration direction making an angle

$$2 \tan^{-1} \left(\frac{b}{a} \right) = 2\theta \text{ with that of the incident light}$$

(Fig. 3).

$$(iii) \text{ If } \delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots \text{ then}$$

$$\cos \delta = 0$$

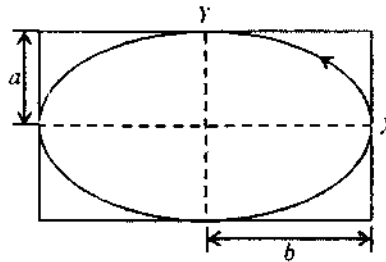
$$\text{and } \sin^2 \delta = 1$$

Then eq. (5) becomes

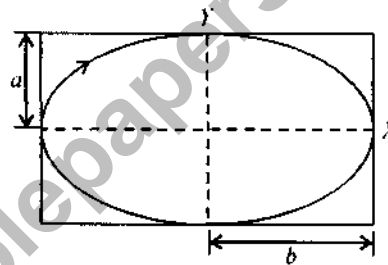
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \dots(8)$$

Hence the emergent light is elliptically polarised.

If $\delta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$, then emergent light is left-handed elliptically polarised.



But if $\delta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \dots$, then emergent light is right-handed elliptically polarised.



$$\text{If } \theta = 45^\circ \text{ then } a = A \cos 45^\circ$$

$$\text{and } b = A \sin 45^\circ$$

Then

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$

$$\text{or } x^2 + y^2 = a^2 \quad \dots(9)$$

Hence in this case the emergent light is circularly polarised.

If $S = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2} \rightarrow$ Left-handed circularly polarised.

If $S = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2} \rightarrow$ Right-handed circularly polarised.

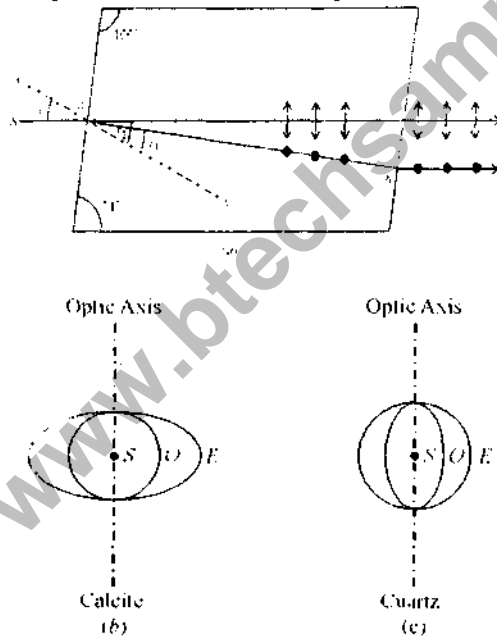
Thus we conclude that the plane polarised and circularly polarised lights are the special cases of elliptically polarised light.

3. (c) Explain the phenomenon of double refraction in calcite or quartz and give the main reason for this phenomenon.

Ans. Double Refraction: When an unpolarised light is passed through a doubly-

refracting crystal such as calcite or quartz, it splits into two refracted rays. One of the rays obeys the ordinary laws of refraction and is called the 'ordinary ray' (O-ray). The other, however, behaves in an extraordinary way and is, therefore, called the 'extraordinary ray' (E-ray). Hence, if an object is viewed through such a crystal, two images of the object are seen, one due to O-ray and the other due to E-ray. The phenomenon is known as 'double refraction'.

Fig. represents the double refraction phenomenon. A ray of light SA from a point source is incident on a calcite crystal making an angle of incidence i , it is refracted along two paths AB and AC making angles of refraction r_1 and r_2 respectively. These rays emerge out along BO and CE parallel to each other. Thus two images O and E can be obtained on the screen. If the crystal is rotated about the incident beam as axis one image O remains stationary while the other image E rotates round the image O .



For ordinary ray the refractive index $\mu_0 = \frac{\sin i}{\sin r_1}$ is constant while for the extraordinary ray $\mu_E = \frac{\sin i}{\sin r_2}$ is not constant but varies with the angle of incidence i .

In calcite crystal $r_2 > r_1$, therefore $\mu_0 > \mu_E$, hence $v_0 < v_E$ that is inside the calcite crystal the E-ray travels faster as compared to O-ray. Such crystals are called uniaxial negative crystals.

In quartz crystal $r_2 < r_1$, therefore $\mu_E > \mu_0$, hence $v_0 > v_E$ that is, inside the quartz crystal the O-ray travels faster as compared to E-ray. Such crystals are called uniaxial positive crystals.

Huygen's Theory of Double Refraction. Huyger explained the phenomenon of double refraction by extending his theory of secondary wavelets in a suitable manner. According to his theory:

(i) Every point on the crystal surface is disturbed by the incident wavefront and becomes the origin of two waveform is one the ordinary and other the extraordinary to account for ordinary and extraordinary rays.

(ii) The wavefront corresponding to the ordinary wavelets is spherical, because O-ray travels with the same velocity in all directions.

(iii) The wavefront corresponding to the extraordinary wavelets is ellipsoidal because E-ray travels with different velocity in different directions.

(iv) The sphere and the ellipsoid touch each other at points which lie on the optic axis of the crystal.

(v) In a negative uniaxial crystal (like calcite) the sphere lies inside the ellipsoidal [Fig. (b)], while in a positive uniaxial crystal (like quartz) the ellipsoid lies inside the sphere [Fig. (c)].

3. (d) **The indices of refraction of quartz for right-handed and left-handed circularly polarised waves of wavelength 7620 \AA travelling in the direction of optic axis have the following values:**

$$\mu_R = 1.53914 \text{ and } \mu_L = 1.53920.$$

Calculate the rotation of the plane of polarization of light in degrees produced by a plate 0.5 mm thick.

Ans. For right-handed substances, the rotation of the plane of polarization is given by the relation

$$\theta_R = \frac{\pi}{\lambda}(\mu_L - \mu_R)d$$

$$\text{Here } d = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

$$\lambda = 7620 \text{ \AA} = 7620 \times 10^{-10} \text{ m}$$

$$\therefore \theta_R = \frac{3.14 \times 0.5 \times 10^{-3}}{7620 \times 10^{-10}} (1.53920 - 1.53914)$$

$$= 0.1236 \text{ rad}$$

$$= \frac{0.2136}{\pi} \times 180 = 7^\circ 5' \quad \text{Ans.}$$

3. (e) Discuss the application of laser in holography and optical communication.

Ans. Applications: The lasers are profitably used in almost every field including fundamental research. The common applications of lasers are as follows:

1. Metallic rods can be melted and joined by means of a laser beam (laser welding).

2. The laser beam is used to vaporise unwanted material during the manufacture of electronic circuits on semiconductor chips.

3. CO₂ gas lasers of about 100 W output are helpful in surgery because they seal small blood vessels while cutting through tissue by vaporising water in the path of their IR beams.

4. Lasers are used to detect and destroy enemy missiles during warfare.

5. Low power semiconductor lasers are used in CD (compact disc) players, laser printers, laser copiers, facsimile machine etc.

6. Semi-conductor lasers are ideal for fibre-optic transmission lines in which the electric signals that would normally be sent along copper wires are first converted into a series of pulses according to a standard code. Lasers then turn the pulses into flashes of IR light that travel along thin glass fibres and at the other end are changed back into electric signals.

7. High power lasers are used to bring about thermonuclear reactions which would become the ultimate inexhaustible power source for human civilization.

8. Lasers are also being employed for separating the various isotopes of an element.

9. Lasers are used in the production of three

dimensional images of an object in holography.

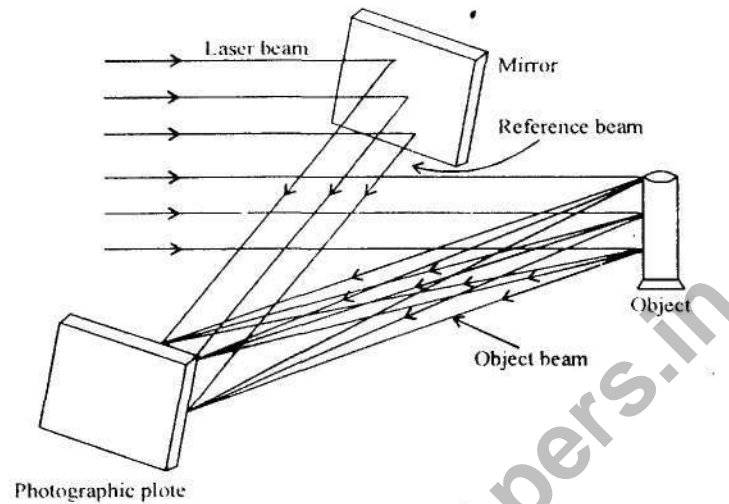
10. Laser beams are also been used in the "inertial confinement" of plasma.

11. The narrow red laser beam is used in supermarkets to read the bar codes.

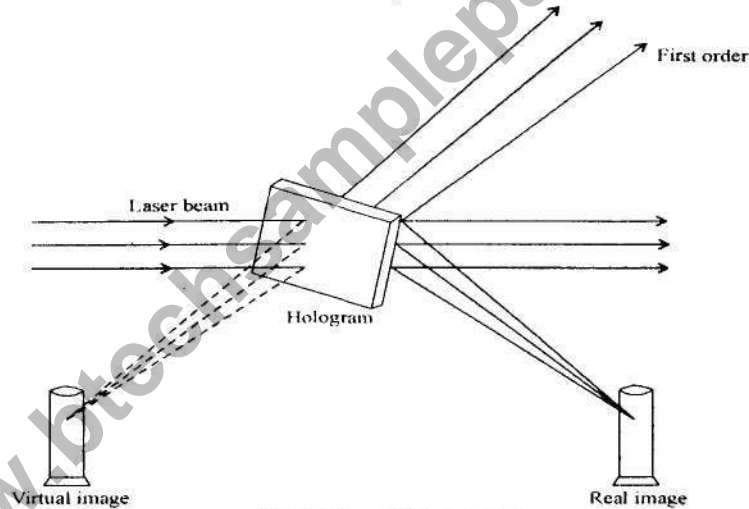
Holography: In 1947, Denis Gabor the English Physicist invented a radically different type of photography, known as holography. He named this new technique wave front construction. According to this technique both the phase and intensity attributes of the wave are recorded and when viewed the photograph shows a three dimensional image of an object. For this work Nobel prize was awarded to Gabor in 1971.

In holography, the light waves reflected from an object are recorded. This photographic record is called a 'hologram'. The hologram has no resemblance with the object, although it contains all information about the object in a kind of optical code. When it is illuminated by a coherent light source, a three-dimensional image of the original object is formed. The formation of image from hologram is known as the *reconstruction process*.

Principle: Fig. illustrates the generation of a hologram. As the light waves reflected from an object are characterised by their amplitude and phase. Gabor recorded the wave-pattern on a photographic plate by mixing two beams namely a reference beam and an object beam (beam reflected by the object). The object is illuminated by highly monochromatic and coherent light (laser beam). Light is reflected from the object to the entire photographic plate. A part of the incident laser beam is made to fall on a plan mirror which reflects it toward the photographic plate. This is the *reference beam*. The object beam reflected from the object interferes with the reference beam and produces an interference pattern on the photographic plate. The photographic plate carrying the interference pattern is called a hologram. This is also known as Gabor zone plate in honour of Denis Gabor who developed the phenomenon of holography.



Generation of a hologram.



Reconstruction process.

In the reconstruction process, the hologram is illuminated by a collimated laser beam which undergoes diffraction phenomenon. One of the diffracted beam emerging from the hologram, when projected back, appears to diverge from an apparent object located in the position of the original object. This beam forms a virtual image which is observed by looking through the hologram. The other beam produces a real image which can be recorded on a photographic plate.

Thus, holography is a two stage process. In the first stage a hologram is recorded in the form of interference pattern. In the second stage the hologram acts as a diffraction grating for the reconstruction beam and the image of the object is reconstructed from the hologram.

Applications:

1. In an ordinary photograph, destruction of a portion may lead to loss of information but in case of hologram each part contains information about the entire object. Therefore a hologram is a reliable source for data storage.

2. Holography is used to detect minute changes that occurred in an object.

3. Holography can be used for testing stresses, strains and surface deformations in objects.

4. By viewing the three dimensional acoustical holograms, in visible light the internal structure of the object can be observed. This technique will be highly useful in the fields of medicine and materials technology.

3. (f) Describe the principle and working of three-level laser system.

Ans. Let us consider an assembly of atoms of some kind that have metastable states of excitation energy $h\nu$. The important requirement to the laser action is that there should be more atoms in the metastable state than in the ground state. If this condition is achieved, there will be more stimulated (induced) emissions from atoms in the metastable state than induced absorptions by atoms in the ground state. The step-by-step process that takes place in laser action are:

Step 1. Atoms in ground state are pumped to state E_2 by photons of energy $h\nu' = E_2 - E_0$, where ν' is the frequency of incident radiation and E_2 , E_0 are the energy of excited state and ground state respectively.

Step 2. Rapid transition of metastable state E_1 by spontaneous emission of photons of energy $h\nu'' = E_2 - E_1$, where ν'' is the frequency of emitted radiation and E_1 is the energy of metastable state.

Step 3. By this way metastable states are occupied in many atoms.

Step 4. Induced emission occurs when photons of energy $h\nu = E_1 - E_0$ are incident with the secondary photons themselves stimulating further transitions to produce an avalanche of coherent beam.

4. Attempt any two of the following:

(10×2)

4. (a) Explain the origin of diamagnetism in materials. Obtain an expression for diamagnetic susceptibility using the Langevin's theory. What is the significance of negative susceptibility?

Ans. Out of Course (According to New Syllabus 2008-09).

4. (b) (i) Discuss the physical significance of Poynting theorem.

(ii) Assuming that all the energy from a 1000 watt lamp is radiated uniformly, calculate the average values of the intensities of electric and

magnetic fields of radiation at a distance of 2 meter from the lamp.

Ans. Out of Course (According to New Syllabus 2008-09).

4. (c) Deduce four Maxwell's equations in free space. Explain the concept of Maxwell's displacement current and show how it led to the modification of Ampere's law.

Ans. Out of Course (According to New Syllabus 2008-09).

5. Attempt any two of the following: (10×2)

5. (a) What is the Compton Effect? Derive the expression for the direction of recoiled Compton electron. Show that the Compton electron can recoil only in the onward direction at angles less than 90° .

Ans. Out of Course (According to New Syllabus 2008-09).

5. (b) (i) Using uncertainty principle show that electron cannot exist in the nucleus.

(ii) Solve Schrodinger's wave equation for a particle in a one dimensional rigid box of side L and having potential energy (V) as follows:

$$V(x) = \infty, \text{ for } x < 0 \text{ and } x > L$$

$$V(x) = 0, \text{ for } 0 \leq x < L$$

Ans. Out of Course (According to New Syllabus 2008-09).

5. (c) (i) What is the physical significance of wave function ψ ?

(ii) Determine the probabilities of finding a particle trapped in a box of length L in the region from $0.45 L$ to $0.55 L$ for the ground state.

Physical constants:

$$\text{Plack's constant } h = 6.63 \times 10^{-34} \text{ J.s}$$

Velocity of light in free space

$$C = 3 \times 10^8 \text{ m/s}$$

$$\text{Electronic charge } e = 1.6 \times 10^{-19} \text{ c}$$

Permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

Permcability of free space

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\text{Rest mass of electron } m_e = 9.1 \times 10^{-31} \text{ kg}$$

Ans. Out of Course (According to New Syllabus 2008-09).