B.TECH. TMA-011

SIXTH SEMESTER THEORY EXAMINATION, 2009-10

GRAPH THEORY

Time: 3 Hours ***Total Marks: 100***

Note: *(i) Attempt all questions.*

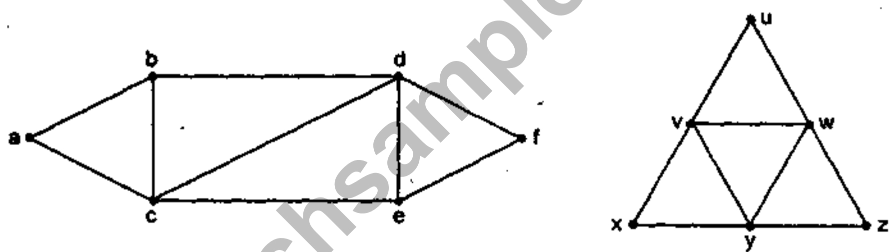
(ii) All questions carry equal marks.

1. Attempt any four of the following: **(5 × 4 = 20)**

(a) Define the degree of a vertex in a graph. Prove that the sum of the degrees of all vertices of a graph in a graph is twice the number of edges in graph.

Ans. Please See Q. 1(a) of 2002–03

(b) Define isomorphism of graphs. For the following pair of graphs, determine whether or not the graphs are isomorphic. Explain your answer.



Ans. Two graphs G_1 and G_2 are said to isomorphism to each other if there is a one-one correspondence between their vertices and edges both so that the incidence relationship is mentained. This given graph is not a isomorphic because in first graph, two vertices have degree two, next two vertices have degree three, next two vertices have degree four where as in the second graph, three vertices have degree two and next three vertices have degree four which is not showing the one to one corresponce between given two graphs.

(c) Prove that a simple graph with *n* vertices and *k* components can have atmost $(n - k)(n - k + 1)/2$ edges.

Ans. Please See Q. 1(c) of 2003–04

(d) Discuss travelling sales man problem.

Ans. Please See Q. 2(b) of 2005–06

(e) Define the following with one example.

(i) Complete graph

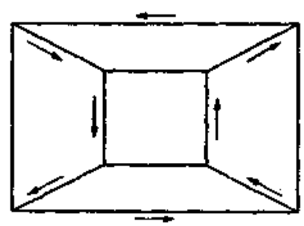
Ans. Please See Q. 1(f) of 2002–03

(ii) Eulerian graph

Ans. Please See Q. 1(c) of 2006–07

(iii) Hamiltonian graph

Ans. Hamiltonian graph: A Hamiltonian graph in a connected graph *G* is defined as a closed trial that traverses every vertex of *G* exactly once except the starting vertex. Example



(iv) Bi-partite graph

Ans. Please See Q. 1(f) of 2002–03

(v) Cut points of a graph

Ans. Please See Q. 1(f) of 2002–03

2. Attempt any four parts of the following: **(4 × 5 = 20)**

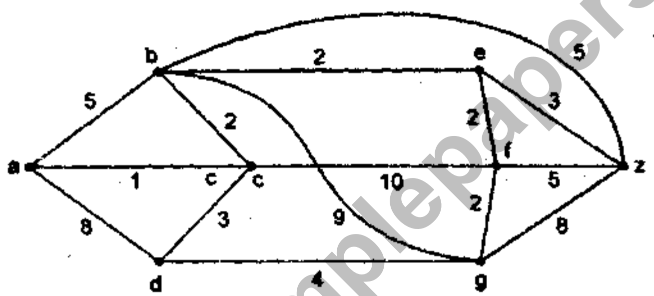
(a) If G is a non-trivial tree, then prove that G contains at least two vertices of degree 1.

Ans. Please See Q. 2(a) of 2002–03

(b) Define binary trees and discuss two important applications of it.

Ans. Please See Q. 2(b) of 2002–03

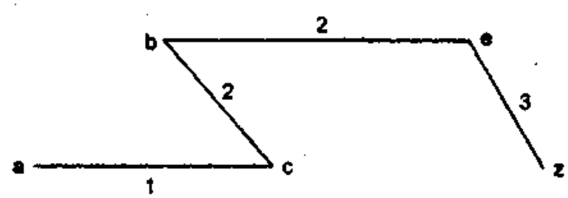
(c) Apply Dijkstra algorithm to find out the shortest path from the vertices a to z in the following graph.



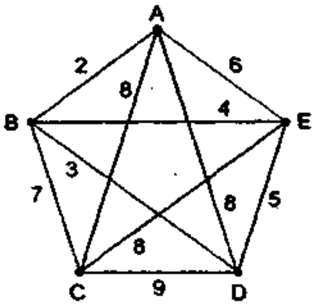
Ans.

Vertex	a	b	c	d	e	f	g	z
Distance	0	∞	∞	∞	∞	∞	∞	∞
	0	5	1	8	∞	∞	∞	∞
	0	3	1	4	∞	11	∞	∞
	0	3	1	4	5	11	12	8
	0	3	1	4	5	11	8	8
	0	3	1	4	5	7	8	8
	0	3	1	4	5	7	8	8

Hence the shortest path from vertices a to z is 8.



(*d*) **Use Prims algorithm to find out the minimal spanning tree of the following graph.**



Ans. Please See Q. 2(d) of 2003–04

(*e*) **Define fundamental circuits. Find the sets of fundamental circuits (four only) of the graph given in Q. No. 2(d). Take any spanning tree and find it corresponding to that spanning tree.**

Ans. Please See Q. 2(f) of 2007–08

(*f*) **Define eccentricity of the vertex and centre of a graph. Find the centre of the graph given in question no. 2(d).**

Ans. The distance from the vertex v to the vertex farthest from v in a graph G is called the eccentricity. It is denoted by $E(v)$

$$E(v) = \max_{v_i \in G} d(v, v_i)$$

Centre of a graph G: A vertex whose eccentricity is the minimum is called centre of the graph G.

The eccentricity of a given graph G is

$$E(A) = 26$$
$$E(B) = 30$$
$$E(C) = 27$$
$$E(D) = 25$$
$$E(E) = 28$$

Hence the centre of a given graph is D.

3. Attempt any four parts of the following:

(4 × 5 = 20)

(*a*) **Define a planar graph. State and prove the Euler's formula for planar graph.**

Ans. Please See Q. 3(C) of 2005–06

(*b*) **Define edge and vertex connectivity of a graph. Prove that the vertex connectivity of any graph will never be more than the edge connectivity.**

Ans. Please See Q. 3(a) of 2007–08

(*c*) **Show that the Kuratowski's first (K_5) and second ($K_{3,3}$) are nonplanar graphs.**

Ans. Please See Q. 3(b) of 2004–05

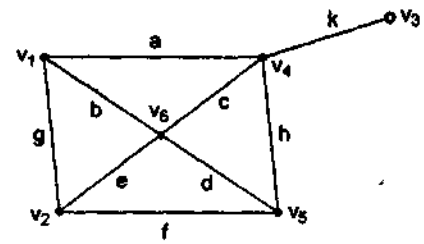
(*d*) **Show that a graph has a dual if and only if it is planar.**

Ans. Please See Q. 3(d) of 2003–04

(*e*) **Define the thickness of a graph, give one example. Find the thickness of Kuratowski's first and second graphs.**

Ans. Please See Q. 3(f) of 2007–08

(*f*) **Define cut-sets. List all cut-sets with respect to the vertex pair v_2, v_3 in the following graph.**



Ans. Cut-set: A cut-set of a connected graph $G = (V, E)$ is a set S ($S \subseteq E$) of edges such that (a) the removal of all edges in S disconnects G and (*b*) the removal of any proper subset of S will not disconnect G.

In a given graph vertex pair V_2, V_3 is not given so it is not possible to find any cut-set with vertex pair V_2, V_3. For detail see, Graph Theory of B. K. Sharma and R. S. Sirohi Book.

4. **Attempt any two parts of the following:**

(2 × 10 = 20)

(a) Define basis vectors of a graph. Show that the number of distinct basis possible in a cut-set subspace is:

$$\frac{1}{r!}(2^r - 2^0)(2^r - 2^1)(2^r - 2^2)\cdots(2^r - 2^{r-1})$$

Ans. Please See Q. 4(a) of 2002–03

(b) Find the relationship among reduced incidence matrix A_f, fundamental circuit matrix B_f and fundamental cut set matrix C_f of a connected graph. Also establish the relation by giving one example.

Ans. Please See Q. 4(b) of 2002–03

(c) (i) If B is a circuit matrix of a connected graph G, with e edges and n vertices, then show that the rank of B is equal to the nullity of G.

Ans. Please See Q. 4(c) of 2005–06

(ii) Prove that the rank of a cut-set matrix is equal to the rank of the graph.

Ans. Let G be a graph of order n. Each row of cut-set matrix can be considered as a vector over $GF(2)$. The no. of is in each column of $A(G)$ is 2. Therefore the sum of all rows under module 2 shall be equal to zero. It means that sum of individual rows of $A(G)$ are not linearly independent.

Consequently rank of $C(G) \leq n - 1$ ...(1)

Again the graph is connected, each vertex of G is adjacent to atleast one edge of G and therefore there will be no row in $C(G)$ sum of whose element is zero.

Now consider the last row of matrix $C(G)$. Again consider a square sub-matrix M of order $(n - 1) \times (n - 1)$ whose rows are those of $C(G)$ after deleting its last row whose columns are those of $C(G)$ including the column C_j. This column C_j shall have the entry 1 in exactly one place only. All the other entries in C_j shall be zero.

Therefore the sum of j^{th} column

$$\alpha_{1,j} C_{1j} + \alpha_{2,j} + \cdots + \alpha_{n-1,j} C_{n-1,j} \neq 0$$

Where scalar $\alpha_{ij} \neq 0$.

This will hold for every column of matrix M. So column of M are linearly independent.

Consequently rank of M is $(n - 1)$. It means that we have got a submatrix M of $C(G)$ whose rank is $n - 1$. Therefore rank of $C(G) \geq (n - 1)$ ...(2)

From (1) and (2)

rank of $C(G) = n - 1$

= which is rank of G.

5. **Attempt any two parts of the following:**

(2 × 10 = 20)

(a) Prove that an m-vertex graph is a tree if and only if its chromatic polynomial is:

$$P_m(\lambda) = \lambda(\lambda - 1)^{m-1}$$

Ans. Please See Q. 5(a) of 2004–05

(b) Show that the number of simple labelled graph of n vertices is $2^{n(n-1)/2}$.

Ans. Please See Q. 5(c) of 2003–04

(c) Define indegree and outdegree of a vertex of a directed graph. Prove that for a directed graph D with n vertices $(v_1, v_2, \ldots v_n)$ and q edges.

$$\sum_{i=1}^{n} \text{in degree}(v_i) = \sum_{i=1}^{n} \text{outdegree}(v_i) = q$$

Ans. Please See Q. 5(b) of 2005–06

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