B. Tech.

FIRST SEMESTER THEORY EXAMINATION 2009-10

ENGG. MECHANICS

Time : 3 Hours **Total Marks : 100**

Note : (i) This paper is in three sections. Section A carries 20 marks, Section B carries 30 marks and Section C carries 50 marks.

(ii) Attempt all questions. Marks are indicated against each question part.

(iii) Assume missing data suitably, if any.

Q. 1. You are required to answer all the parts : **$2 \times 10 = 20$**

Choose correct answer for the following parts :

(a) If number of forces act simultaneously on a particle, it is possible :

(i) not to replace them by a single force (ii) to replace them by a single force

(iii) to replace them by a single couple (iv) to replace them by a force and couple

(b) Moment of intertia of a circular area, about an axis perpendicular to the area passing through its centre is given by :

(i) $\frac{\pi d^4}{8}$ (ii) $\frac{\pi d^4}{16}$ (iii) $\frac{\pi d^4}{32}$ (iv) $\frac{\pi d^4}{64}$

Ans. (a) → (ii) to replace them by a single force. (b) → (iii) $\frac{\pi d^4}{32}$

Fill in the blanks for the following three parts :

You will be awarded full marks, if all the entries in a part are correct otherwise will be awarded zero.

(c) In truss analysis, the weight of truss member is assumed to be .... and stress induced on application of force in truss members is ....

(d) Centripetal component of acceleration is measured .... to the direction of velocity and acts along a line .... to the path of rotation and towards the centre of curvature of path.

(e) The value of shear stress which is induced in the shaft due to the applied torque is .... at the centre and .... at the circumference.

Ans. (c) negligible , axial; (d) perpendicular, normal; (e) zero maximum/highest

Match the columns for the following three parts :

You will be awarded full marks, if all the matches in a part are correct otherwise will be aworded zero.

(f)

	Column-I		Column-II
(i)	Statics	(P)	Study of forces that causes motion
(ii)	Dynamics	(Q)	Study of forces in rigid bodies
(iii)	Kinetics	(R)	Study of displacement, velocity and acceleration
(iv)	Kinematics	(S)	Study of forces in moving bodies

(g)

	Column-I		Column-II
(i)	Lami's theoren	(P)	Dynamic equilibrium of particle

(ii)	Maxwell theorem	(Q)	Principle of moments
(iii)	D' Alembert's principle	(R)	Equilibrium of three concurrent forces
(iv)	Varignon's theorem	(S)	Force analysis of trusses

(h)

	Column-I		Column-II
(i)	Torsional rigidity	(P)	EI
(ii)	Section modulus	(Q)	GJ
(iii)	Torsional stiffness	(R)	$\frac{I}{y}$
(iv)	Flexural rigdity	(S)	$\frac{T}{\theta}$

Ans. (f) : (i) → (Q), (ii) → (S), (iii) → (P), (iv) → (R)

(g) : (i) → (R), (ii) → (S), (iii) → (P), (iv) → (Q)

(h) : (i) → (Q), (ii) → (R), (iii) → (S), (iv) → (P)

Choose the correct answer for the following two parts :

(i) Two forces can be in equilibrium only if they are :

(I) equal in magnitude

(II) opposite in direction

(III) collinear in action

(i) Only I and II are correct (ii) Only I and III are correct

(iii) Only II and III are correct (iv) All are correct

(j) For the same power transmitted :

(I) the weight of solid shaft is less than that of the hollow shaft

(II) the weight of hollow shaft is less than that of the solid shaft

(III) No relation exists between power transmitted and the weight of solid and hollow shaft

(i) Only I and II are correct (ii) Only II and III are correct

(iii) II alone is correct (iv) I alone is correct

Ans. (i) → (iv) All are correct

(j) → (iii) II alone is correct

Q. 2. Answer any three parts of the following : $10 \times 3 = 30$

(a) Three cylinders A, B and C each weighing 100 N and diameter 80 mm are placed in a channel of 180 mm width as shown in Fig. 1. Determine the pressure exerted by the cylinder A and B at the point of contact.

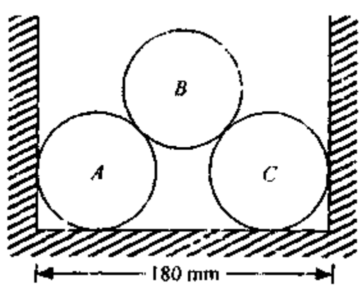
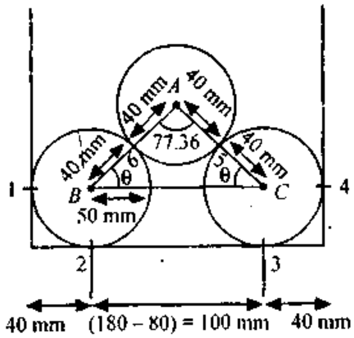


Fig. 1.

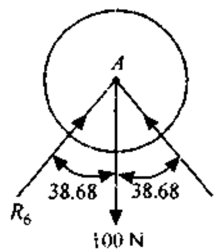
Ans. (a)

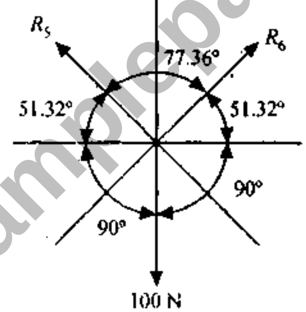


1, 2, 3, 4, 5, 6 are point of contacts $\theta = \cos^{-1}\frac{50}{80} = 51.32°$

$$\angle BAC = 180° - 51.32 \times 2 = 77.36°$$

F.B.D. of ball A :



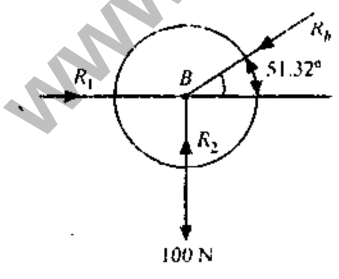


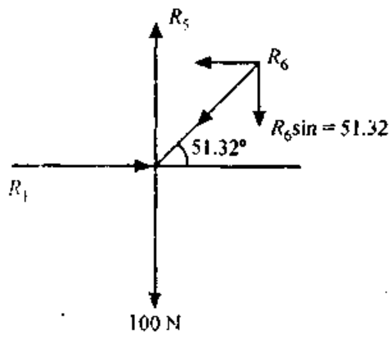
From Lami's theorem $\frac{R_5}{\sin(90+51.32)} = \frac{R_6}{(90+51.32)} = \frac{100}{\sin 77.36}$

$\therefore$ $R_5 = 62.49$ N

$R_6 = 62.49$ N

F.B.D. of ball B :





$(\xrightarrow{T})$

$\therefore$ $\Sigma(F_H) = R_1 - R_6 \cos 51.32°$

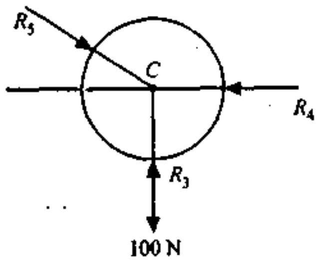
$\therefore$ $R_1 = 39$ N

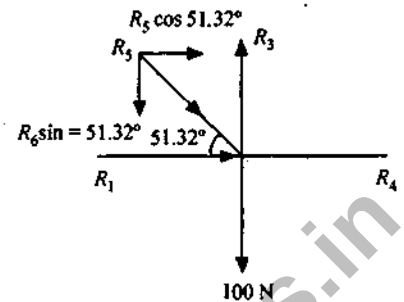
$(T\uparrow)\Sigma F_v = 0$

$\therefore$ $R_2 = 100 - R_6 \sin 51.32°$

$\therefore \quad R_2 = 148.78\,N$

F.B.D. of Ball C :





$$(\xrightarrow{T})$$

$$\Sigma F_H = 0$$

$$R_5 \cos 51.32 - R_4 = 0$$

$$R_4 = 39\,N$$

$$(+\uparrow \Sigma F_v) = 0$$

$\therefore \quad R_5 \sin 51.32 + 100 - R_3 = 0$

$$R_3 = 148.78\,N$$

Pressure at point of contact of *A* and *B* are

at point (1) = 39 N, at point (2) = 148.78 N, at point (3) = 148.78 N

at point (4) = 39 N, at point (5) = 62.49 N, at point (6) = 62.49 N

(b) Calculate the values of shear force and bending moments for the cantilever beam shown in Fig. 2. Also draw the shear force and bending moment diagrams.

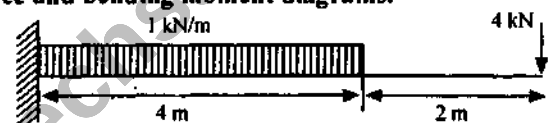
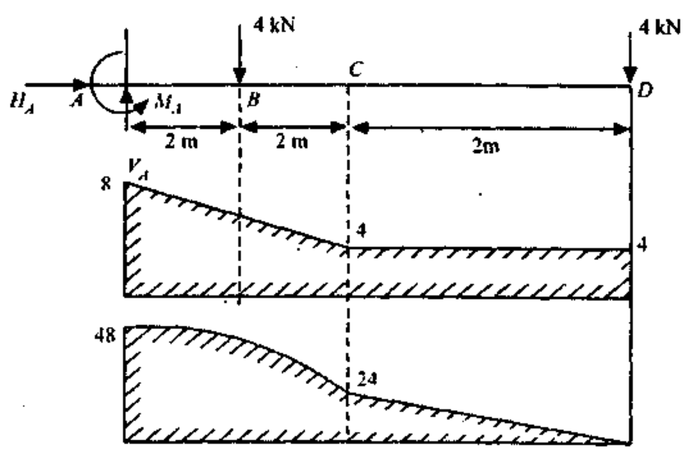


Fig. 2.

Ans.

Fig. 2.



Taking moment about A

$$M_A - 4\times2 - 4\times8 = 0$$
$$M_A = 48\text{kN.m}$$

Taking the equlibrium condition

$$V_A - 4 - 4 = 0$$

$\therefore$
$$V_A = 8\text{kN}$$
$$H_A = 0$$

For S.F. diagram calculation

S.F. between $AB = 4$ kN

S.F. between $BC = 8 - 4 = 4$ kN

S.F. between $DC = 4 - 4 = 0$

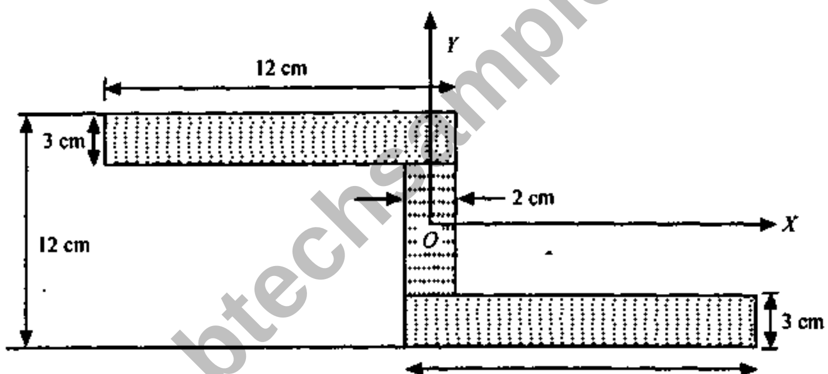
B.M. Calculation

B.M. at point $A = 48$kN.m

B.M. at point $C = 48 - 8\times4 + 4\times2 = 24$ kN.m

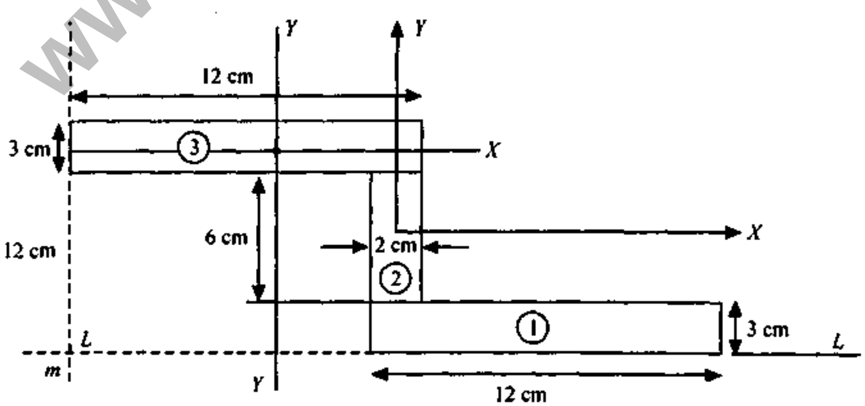
B.M. at point $D = 48 - 8\times8 + 4\times4 = 0$

(c) For the z-section as shown in Fig., the moment of inertia with respect to x and y axes are given as $I_x = 1548\text{ cm}^4$ and $I_y = 2668\text{ cm}^4$. Determine the principal axes of the section about O (centroid of vertical section and point O coincides) and values of the principal moments of inertia.



Fog. 3.

Ans.



Components	Area (a)	Centroidal distance x	Centroidal distance y	$ax(mm)^3\ a$	$ay(mm)^3$
(1)	$3\times12=36$	$10+6=16$	1.5	$3\times12\times16=576$	$3\times12\times1.5=54$
(2)	$6\times2=12$	$10+1=11$	6	$6\times2\times11=132$	$6\times2\times6=72$
(3)	$30\times12=360$	6	$1.5+9=10.5$	$30\times12\times6=2160$	$30\times12\times10.5=3780$

$$\bar{x}=\frac{\Sigma ax}{a}=\frac{576+132+2160}{36+12+360}=7.03\ \text{cm}$$

$$\bar{y}=\frac{\Sigma ay}{a}=\frac{54+72+3780}{36+12+360}=9.57\ \text{cm}$$

Poincipal axis is $x-x$ and $y-y$

$$I_{xx}=1548+(36+12+360)(7.03-6)^2=1980.85\ \text{cm}^4$$

$$I_{yy}=2668+(36+12+360)(11-9.57)^2=3502.32\ \text{cm}^4$$

(d) A horizontal bar 1.5 m long and of small cross section rotates about vertical axis through one end. It accelerates uniformly from 1200 rpm to 1500 rpm in an interval of 5 seconds. What is the linear velocity at the beginning and end of the interval? What are the normal and tangential components of acceleration of the mid point of the bar after 5 seconds after the acceleration begins?

Ans. Initial angular velocity 1200 r.p.m.

$\therefore$ $$\omega_0=\frac{1200}{60}\ \text{rev/sec}=20\ \text{rev/sec}$$

Final angular velocity 1500 r.p.m.

$\therefore$ $$\omega=\frac{1500}{60}\ \text{rev/sec}=25\ \text{rev/sec}$$

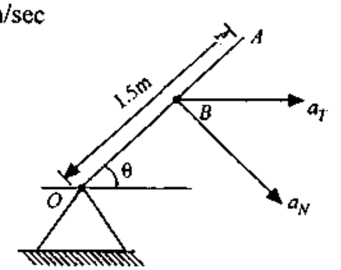
Time taken during constant acceleration $(t_1)=5\text{sec}$

$$r=1.5\ \text{m}$$

$\therefore$ Initial velocity $\quad v_0=r\omega_0=1.5\times20=30\ \text{m/sec}$

Final velocity $\quad v=r\omega=1.5\times25=37.5\ \text{m/sec}$

at mid point normal and tangential acceleration are a_N and a_T



Assume $\quad \theta=36.87°$

$$a_T=r\alpha=1.5\alpha$$

Horizontal component of velocity (v) 37.5 m/sec is

$$V=v\cos(90-\theta)$$

$\therefore$ $$V=v\sin\theta$$

$$37.5=v\sin\theta$$

$$v=62.5\ \text{m/sec}$$

Normal conponent of acceleration

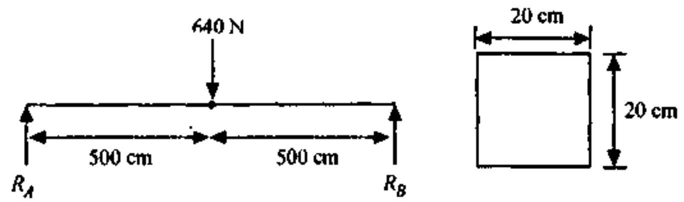
$$a_n=\frac{(62.5)^2}{1.5}=2604.17\ \text{m}^2/\text{sec}$$

$$62.5=-a_N\sin\theta+a_T\sin(90-\theta)=-2604.17\sin 36.87°+a_t\cos 36.87°$$

$$a_T=2031.26\ \text{m}^2/\text{sec}$$

(e) A cast iron test beam 20 mm × 20 mm in section and 1 m long and supported at the ends fails when central load of 640 N is applied. What uniformly distributed load will break a cantilever of the same material 50 mm wide, 100 mm deep and 2 m long?

Ans.



$b = 20\text{cm},\ d = 20$

$$I = \frac{bd^3}{12} = \frac{20 \times 20^3}{12} = 13333.33\ \text{mm}^2$$

$$\text{(moment)}\ M = R_A \times 500 = 320 \times 500 = 160{,}000\ \text{N.mm}$$

By the using of equilibrium condition support reaction R_A and R_B are

$$R_A + R_B = 640$$

$$\Sigma m_A = 0$$

$\therefore$ $$R_A \times 0 - 640 \times 500 + R_B \times 1000 = 0$$

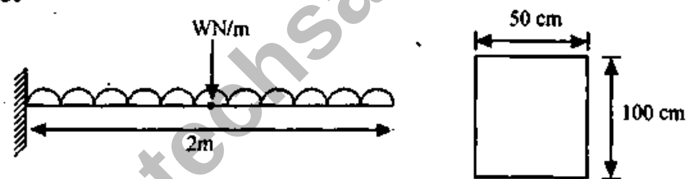
$$R_B = 0.320\ \text{N}$$

$\therefore$ $$R_A = 320\ \text{N}$$

$$\frac{M}{I} = \frac{\sigma_0}{y}$$

$\therefore$ $$\sigma_0 = \frac{My}{I} = 160{,}000 \times \frac{20}{2} \times \frac{1}{13333.33} = 120\ \text{N/mm}^2$$

Now for 2nd case



$b = 50\text{mm},\ d = 100\text{mm}$

$\therefore$ $$I = \frac{bd^3}{12} = \frac{50 \times 100^3}{12} = 4166666.667\ \text{mm}^4$$

$$M = \frac{WL^2}{2}$$

(maximum bending moment of contilever under uniformely distributed coad)

$$M = \frac{W \times (2 \times 10^3)^2}{2}$$

$$\sigma_b = 120\ \text{N/mm}^2$$

$$y = \frac{100}{2} = 50\ \text{mm}$$

$\therefore$ $$\frac{M}{I} = \frac{\sigma_b}{y}$$

$\therefore$ $$\frac{W \times (2 \times 10^3)^2}{2} \times \frac{1}{4166666.667} = \frac{120}{50}$$

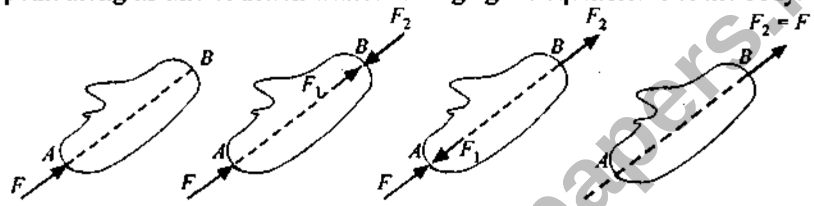
$$W = 5\ \text{N/m}$$

$\therefore$ Uniformely distributed load require will be 5 N/m

Q. 3. Answer any two Parts of the following : $4 \times 2 = 10$

(a) Explain the theorem of transmissibility of a force. What are its limitations ?

Ans. The principle of transmissibility of force states that shifting of the point of application of a force acting on a body to any other point on the like of action of the force without changing its direction causes no change in the equilibrium state of the body *i.e.*, the condition of motion of the body remains unchange for example a force F is acting at point A on a rigid body along the line of action AB. At point B apply two equal and opposite forces F_1 and F_2 such that F_1 and F_2 are collinear and equal in magnitude with F. Now we contransfer F_1 from B to A such that F and F_1 are equal and opposite and accordingly they cancel each other. The net result is force F_2 at B. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing the equilibrium of the body.



Limitations :

The above principle is valid only for rigid bodies.

Body should remain its shape size so no internal forces develop on the body.

Sole effect of the force is on the state of motion of the body only.

(b) Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 rpm. The coefficient of friction between belt and pulley is 0.25, angle of lap 160° and maximum tension in the belt is 2.5 kN.

$d = 600\,\text{mm} = 0.6\,\text{m},\ N = 200\,\text{rpm},\ M = 0.25$

$$\theta = 160° = 160 \times \frac{\pi}{180} = 2.79\,\text{rad}$$

$$T_1 = 2.5\,\text{KN}$$

$$\text{speed of the bell} = \frac{\pi d N}{60} = \frac{z \times 0.6 \times 200}{60} = 6.28\,\text{rad/sec}$$

$$2.3 \log_{10}\left(\frac{T_1}{T_2}\right) = \mu\theta = 0.25 \times 2.79 = 0.6975$$

$$\log\left(\frac{T_1}{T_2}\right) = 0.3033$$

$$\log\left(\frac{2.5}{T_2}\right) = 0.3033$$

$$T_2 = 1.24\,\text{kN}$$

Power transmitted by the bell

$$P = (T_1 - T_2)^v = (2.5 - 1.25) \times 6.28$$

$$= 12.56\,\text{kW}$$

(c) Fig. shows a system of levers supporting a load of 500 N. Determine the reactions at the supports A and B.

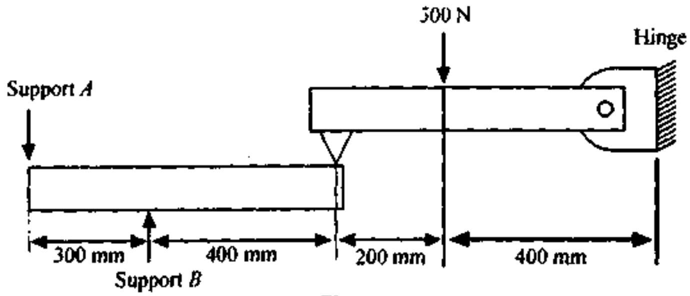
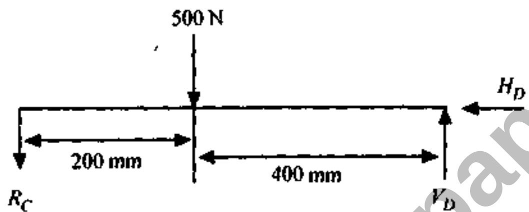


Fig.

Ans. For Ist protion

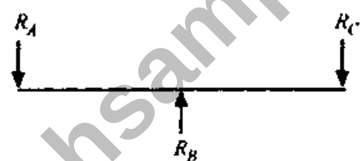


Using the equlibrium condition

$$\Sigma F_H = 0 = H_D = 0$$

$$\Sigma F_V = R_C + V_D = 500$$

For IInd Part



$$\Sigma F_v = 0$$

$\therefore$ $$R_A + R_C = R_B$$

Taking the moment about D

$$500 \times 400 + R_C \times 600 = 0$$

$$R_C = -333.33 \text{ M}$$

Taking the moment about A

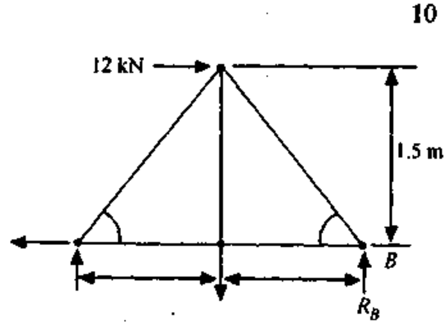
$$R_A \times 0 + R_B \times 300 - R_C + 700 = 0$$

$\therefore$ $$R_B = \frac{333.33 \times 700}{300} = 777.78 \text{ N}$$

$\therefore$ $$R_A = R_B - R_C = 1111.07 \text{ N}$$

Q. 4. Answer any one part of the following : **10**

(a) Find the force in all members of a truss as shown in Fig. 5 which carries a horizontal load of 12 kN at point *D* and vertical load of 18 kN at point *C*.



Ans.

1.	F_{AC}	15 kN	T
2.	F_{AD}	3.75 kN	C
3.	F_{BC}	21 kN	T
4.	F_{BD}	26.25 kN	C
5.	F_{CD}	18 kN	T

Taking moment about A,

$$-18\times 2+R_B\times 4-18\times 1.5=0$$

$$R_B = 15.75\,\text{kN}$$

$(+\uparrow)\Sigma F_v = 0$ $\quad R_{AY} = 18+R_B = 0$

$\therefore$ $\quad R_{AY} = 18-15.75 = 2.25\,\text{kN}$

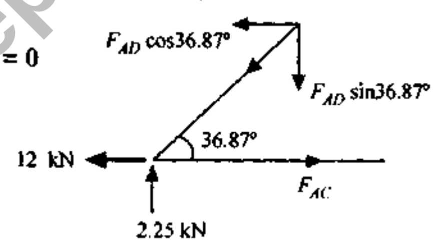
$$\theta = \tan^{-1}\frac{1.5}{2} = 36.87^\circ$$

$\Sigma F_H = 0$

$$R_{AX} - 12 = 0$$

$\therefore$ $\quad R_{AX} = 12\,\text{kN}$

Consider the joint A :



$(+\uparrow)\Sigma F_v = 0$ $\quad 2.25 = F_{AD}\sin 36.87^\circ = 0$

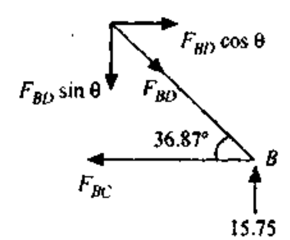
$\therefore$ $\quad F_{AD} = 3.75\,\text{kN}$

$(\xrightarrow{+})\ \Sigma F_H = 0$

$$-12+F_{AC}-F_{AD}\cos 36.87^\circ = 0$$

$\therefore$ $\quad F_{AC} = 15\,\text{kN}$

Consider the joint B :

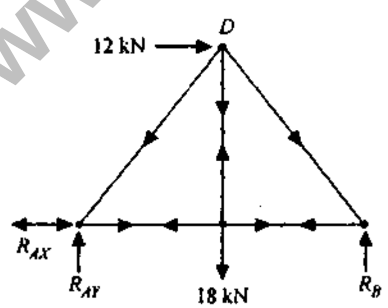


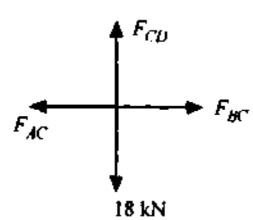
$(+\uparrow)\Sigma F_v = 0$ $\quad 15.75 - F_{BD}\sin 36.87^\circ = 0$

$$F_{BD} = 26.25\,\text{kN}$$

$(\xrightarrow{+})\ \Sigma F_H = 0$ $\quad F_{BD}\cos - F_{BC} = 0$

$\therefore$ $\quad F_{BC} = 26.25\cos 36.87 = 21\,\text{kN}$





Consider joint C :

$\therefore$ $\quad F_{CD} = 18\,\text{kN}$

(b) A beam is loaded as shown in Fig. 6. Draw its shear force and bending moment diagram.

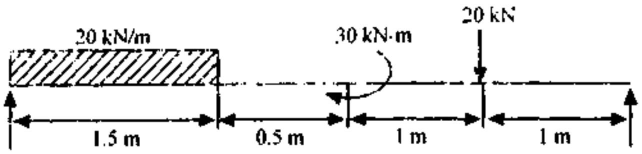
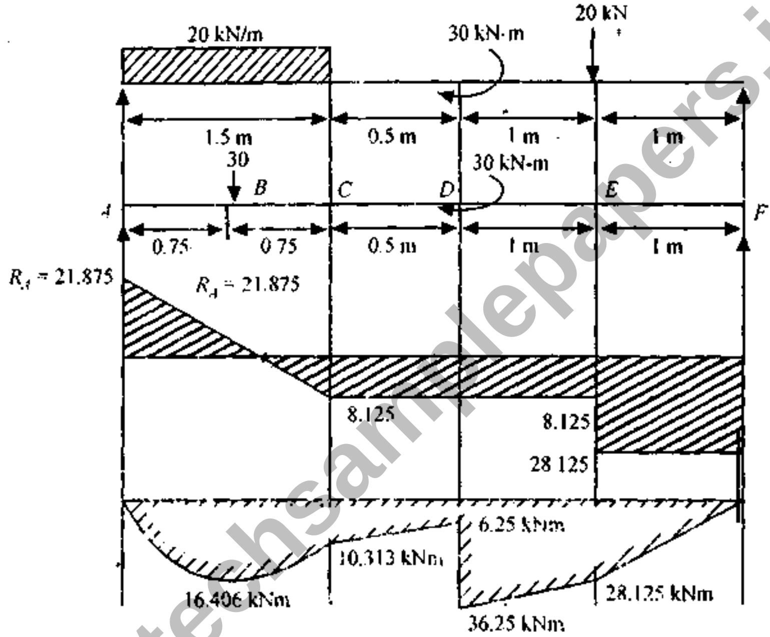


Fig. 6.

Ans.



Using equlibrium condition

$$\Sigma V_F = 0$$

$$R_A + R_F = 30 + 20 = 50$$

Moment about A

$$\Sigma M_A = 0$$

$$-30 \times 0.75 - 30 - 20 \times 3 + R_F \times 4 = 0$$

$\therefore$

$$R_F = 28.125 \text{ kN}$$

$$R_A = 21.875 \text{ kN}$$

Calculation for S.F. diagram

S.F. between $BA = 21.875$ kN

S.F. between $CB = 21.875 - 30 = -8.125$ kN

S.F. between $CE = -8.125$ kN

S.F. between $EF = -8.125 - 20 = -28.125$ kN

S.F. at $F = -28.125 + 28.125 = 0$

B.M. diagram

B.M. at point $A = 0$

B.M. at point $B = -21.875 \times 0.75 = -16.406$ kN-m

B.M. at point $C = -21.875 \times 1.5 + 30 \times 0.75 = -10.313$ kN-m

B.M. at point $D = -21.875 \times 2 + 30 \times 1.25 - 30 = -36.25$ kN-m

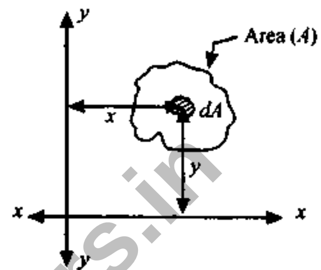
B.M. at point $E = -21.875 \times 3 + 30 \times 2.25 - 30 = -28.125$ kN-m

B.m. at point $F = -21.875 \times 4 + 30 \times 3.25 - 30 + 20 = 0$

Q. 5. Answer any two parts of the following : **$5 \times 2 = 10$**

(a) Explain the following : (i) Product of inertia (ii) Principal moment of inertia

Ans. (i) Product of inertia : Consider a place area or section (area A) as shown in figure. Further consider an element area dA at a distance x and y from yy axis and xx axis respectively. Then $\Sigma xy dA$ is defined as the product of inertia of cross section.



Mathmetically $I_{xy} = \Sigma xy\, dA = \int_A xy\, dA$

(ii) Principal moment of Inertia : If the two areas about which the product of inertia found are such that the product of inertia becomes zero, the two axes are then called the principal axes. The moment of inertia about a principal axis is called the 'principal moment of inertia'.

(b) A semicircular area is removed from the trapezoid as shown in Fig. 7. Determine the centroid of remaining area :

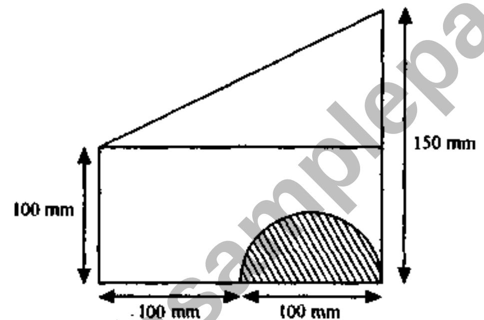
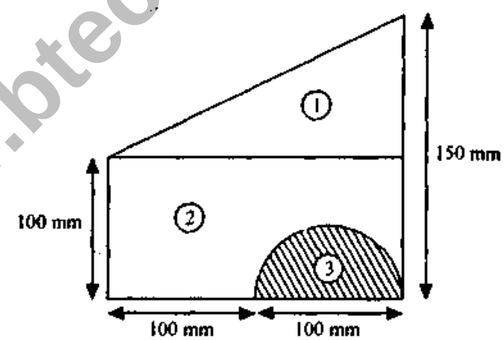


Fig. 7.

Ans.



Total figure has been divided in 3 basic figure (1) is triangle, (2) is rectangle and (3) is semicircle.

(1) Triangle $a_1 = \dfrac{(100+100) \times (150-100)}{2} = 5000\,\text{mm}^2$

$y_1 = 100 + \dfrac{50}{3}\,\text{mm} = 116.67\,\text{mm}$

$x_1 = 200 \times \dfrac{2}{3} = 133.33\,\text{m}$

(2) Rectangle $a_2 = 100 \times 200 = 20000\,\text{mm}^2$ $y_2 = \dfrac{100}{2} = 50\,\text{mm}$ $x_2 = \dfrac{200}{2} = 100\,\text{mm}$

(3) Semicircle $a_3 = \frac{\pi}{2} \times (50)^2 = 3927 \text{mm}^2$ $y_2 = \frac{4 \times 50}{3\pi} = 21.22 \text{mm}$ $x_2 = 100 + \frac{100}{2} = 150 \text{mm}$

$$\therefore \quad \bar{x} = \frac{a_1x_1 + a_2x_2 - a_3x_3}{a_1 + a_2 - a_3} = \frac{5000 \times 133.33 + 20{,}000 \times 100 - 3927 \times 150}{5000 + 20{,}000 - 3927} = 98.59 \text{mm}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 - a_3y_3}{a_1 + a_2 - a_3} = \frac{5000 \times 116.67 + 20{,}000 \times 50 - 3927 \times 21.22}{5000 + 20{,}000 - 3927} = 71.18 \text{mm}$$

(c) Derive an expression of mass moment of inertia of a circular lamina about the central axis.

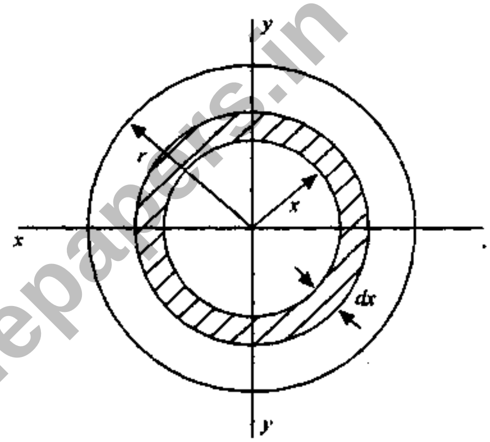
Ans. Consider a circle $ABCD$ of radius (r) with centre O and x-x' and y-y' be two axes of reference through O.

Now consider an elementry ring of radius x and thickness dx therefore area of the ring.

$$da = 2zx\, dx$$

and moment of inertia of ring about x-x axis and y-y axis

$$= \text{Area} \times (\text{distance})^2 = 2zx\, dx \times x^2 = 2zx^3\, dx$$



Now moment of inertia of the whole section about the centre axis

$$I_{zz} = \int_0^r 2zx^3\, dx = 2z\int_0^r x^3\, dx = 2z\left[\frac{x^4}{4}\right]_0^r$$

$$= \frac{\pi}{2}(r^4) = \frac{z}{32}d^4 \left[\text{as} \quad r = \frac{d}{2}\right]$$

$$I_{xx} + I_{yy} = I_{zz}$$

$$\therefore \quad I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{1}{2}\frac{\pi}{32}(d^4) = \frac{z}{64}(d^4)$$

Q. 6. Ans. any one of the following : **10**

(a) A cord is wrapped around a wheel of radius 0.2 m, which is initially at rest as shown in Fig. 8. If a force is applied to the cord and gives it as acceleration $a = (4t)$ m/sec^2, what t is in second. Determine the angular velocity of the wheel and the angular position of line OP both as a function of time.

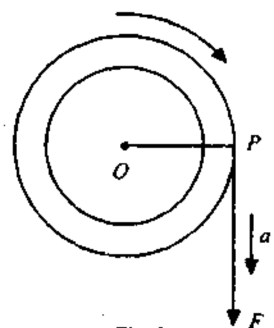


Fig. 8.

Ans. (a) $a = 4t$

$a = r\omega^2$ $\therefore$ $0.2(\alpha) = 4t$

$$\alpha = \frac{d\omega}{dt} = 20t$$

Integrating $\int_0^\omega d\omega = 20\int_0^t t\, dt$

$$\omega = 10t^2 \text{ rad/ sec}$$

$$\omega = \frac{d\theta}{dt} = 10t^2$$

Integrating $\int_0^\theta d\theta = 10\int_0^t t^2\, dt$

$r\alpha = 4t$ $\therefore$ $\alpha = 20t$ (rad/sec^2)

$d\omega = 20dt$

$$\omega = 20\left[\frac{t^2}{2}\right]_0^t$$

$\therefore$ $d\theta = 10t^2\ dt$ $\therefore$ $\theta = \frac{10t^3}{3}$ rad

$\therefore$ angular velocity $\omega = \frac{d\theta}{dt} = 10t^2$

angular position $\theta = \frac{10t^3}{3}$ rad

6. (b) A road roller has a total mass of 12000 kg. The front roller has a mass of 2000 kg, a radius of gyration of 0.4 m and a diameter of 1.2 m. The rear axle, together with its wheels, has a mass of 2500 kg, a radius of gyration of 0.6 m and a diameter of 1.5m. Calculate kinetic energy of rotation of the wheels and axles at a speed of 9 km/h and total kinetic energy of road roller.

Ans. $m = 12000$ kg, $m_1 = 2000$ kg, $k_1 = 0.4$ m, $d_1 = 1.2$ m

$r_1 = 0.6$ m, $m_2 = 2.5t = 2500$ kg $k_2 = 0.6$ m, $d_2 = 1.5$ m,

$r_2 = 0.75$ m $v = 9$ km/h $= 2.5$ m/sec $S = 6$ m

1. Kinetic energy of rotation of the wheel and axles mass moment of inertia of the front roller

$$I_1 = m_1(k_1)^2 = 2000(0.4)^2 = 320\ \text{kg-m}^2$$

mass moment of inertia of the rear axle together when its wheels

$$I_2 = m_2(k_2)^2 = 2500(0.6)^2 = 900\ \text{kg-m}^2$$

Angular speed of the front roller

$$\omega_1 = \frac{v}{r_1} = \frac{2.5}{0.6} = 4.16\ \text{rad/sec}$$

Angular speed of rear wheels $\omega_2 = \frac{v}{r_2} = \frac{2.5}{0.75} = 3.3$ rad/sec

Kinetic energy of rotation of front roller

$$E_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}\times 320\times(4.16)^2 = 2770\ \text{N-m}$$

Kinetic energy of rotation of rear axle together with its wheel

$$E_2 = \frac{1}{2}I_2\omega_2^2 = \times\frac{1}{2}\times 900\times(3.3)^2 = 4900\ \text{N-m}$$

Total kinetic energy $E = E_1 + E_2 = 2770 + 4900 = 7670$ N-m

2. Total kinetic energy of motion of road roller

$$E_3 = \frac{1}{2}mv^2 = \frac{1}{2}\times 12000\times(2.5)^2 = 37500\ \text{N-m}$$

The energy include kinetic energy of translation of the wheel also because the total mass (m) has been considered.

$\therefore$ Total kinetic energy of road roller (E_n) = kinetic energy of transtation

+ kinectic energy of rotation

$= E_3 + E = 37500 + 7670 = 45170$ N-m

Q. 7. Answer any one of the following : **10**

(a) Write the assumptions made in the theory of simple bending.

A beam of I-section is 250 mm deep. The flanges are 15 mm thick, 100 mm wide while the web is 8 mm thick. Compare the flexural strength of this beam section with a rectangular section of the same material and area whose width is two-third depth.

Ans. (a) Assumption is simple bending theory :

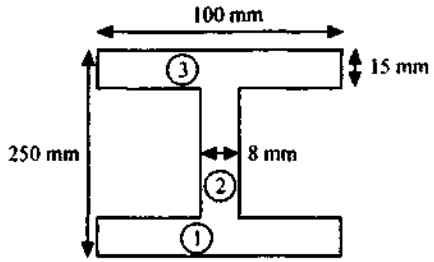
(i) The material is homogeneous isotropic and has the same value of modulus of elasticity for compression and tension.

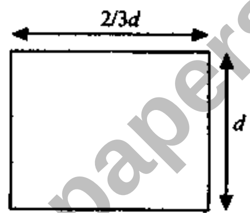
(ii) The beam material is stressed with in the elastic limit and thus follows Hooke's law.

(iii) The transverse cross section remain plane and perpendicular to the neutral surface after bending.

(iv) The beam is initially straight and all longitudinal filaments bend into circular area with is common centre of curvatur.

(v) The plane of loading must contain a principal axis of the beam cross section and the loads must be perpendicular to the longitudinal axis of the beam.





For rectangular section :

$$I = \frac{\frac{2}{3}d \times d^3}{12} = \frac{2d^4}{36}$$

$$y = \frac{d}{2}$$

$$\frac{E}{R} = \frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_R = \frac{My}{I} = \frac{M \times \frac{d}{2}}{\frac{2d^4}{36}} = \frac{Md}{2} \times \frac{18}{d^4} = \frac{9m}{d^3}$$

For I section :

$$A_1 = 100 \times 15 = 1500\,\text{mm}^2 \qquad y_1 = 7.5\,\text{mm}$$

$$A_2 = 8 \times (250 - 30) = 1760\,\text{mm}^2 \qquad y_2 = 15 + \frac{220}{2} = 125\,\text{mm}$$

$$A_3 = 100 \times 15 = 1500\,\text{mm}^2 \qquad y_3 = \frac{15}{2} + 220 + 15 = 242.5\,\text{mm}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{1500 \times 7.5 + 1760 \times 125 + 1500 \times 242.5}{1500 + 1760 + 1500} = 125\,\text{mm}$$

$$\therefore \qquad \sigma_I = \frac{My}{I} = \frac{M \times 125}{4.6326 \times 10^{10}}$$

$$I = I_{xx_1} + I_{xx_2} + I_{xx_3} = \frac{100 \times 15^3}{12} + 100 \times 15(125 - 7.5)^2 + \frac{8 \times 220^3}{12} + 8 \times 220$$

$$\times (125 - 125)^2 + \frac{100 \times 15^3}{12} + 100 \times 15(242.5 - 125)^2$$

$$= 4.6326 \times 10^{10} \text{ mm}^4$$

$$\therefore \quad \frac{\sigma_I}{\sigma_R} = \frac{M \times 125}{4.6326 \times 10^{10}} \times \frac{d^3}{gM} = \frac{13.89 d^3}{4.6325 \times 10^{10}}$$

Q.7. (b) Prove that shear stress due to pure torsion is directly proportional to the radius of the shaft. The average torque transmitted by a shaft is 2255 Nm. The maximum torque is 146% of average torque. If the allowable shear stress in the shaft material is 45 N/mm², determine the suitable diameter of the shaft.

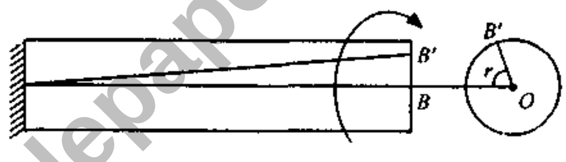
Ans. (b) Consider a shaft of Radius R, length L is subjected to a torque T on the free end and other end is fixed.

Initially the line of shaft is horizontal AB before twisting and after twisting it takes position AB'

Angle of twist $\angle BAB' = \phi$, Shear strain $\angle BOB' = \theta$

From longitudinal section $\tan\phi = \frac{BB'}{L}$, $BB' = \angle\phi$

From cross section length of are = Radius X angle



$$\because \quad \tan\theta = \theta$$

$$BB' = r\theta$$

$$\angle\phi = r\theta$$

$$\phi = \frac{r\theta}{2}$$

Shear modulur $G = \frac{\text{shear strear}}{\text{shear strain}} = \frac{\tau}{\phi}$

$$\therefore \quad \phi = \frac{\tau}{G}$$

$$\therefore \quad \frac{\tau}{G} = \frac{r\theta}{L}$$

$$\therefore \quad \tau \propto r \quad \text{when } G, \theta, L \text{ are constant}$$

$$T_{avg} = 2255 \text{ N-m}$$

$$T_{avg} \times 1.146 = \tau_{max}$$

$$\tau = 45 \text{ N/mm}^2$$

$$d = ?$$

$$T_{max} = 2255 \times 1.146 = 2584.23 \text{ N.m} = 2584.23 \times 10^3 \text{ N.mm}$$

$$\frac{T}{\tau} = \frac{\tau}{R} \qquad \tau = \frac{zd^4}{32}$$

$$\therefore \quad \frac{2584.23 \times 10^3}{\frac{\pi d^4}{32}} = \frac{45}{\frac{d}{2}}$$

$$\therefore \quad \frac{2584.23 \times 10^3 \times 32}{zd^4} = \frac{45 \times 2}{d}$$

$$\therefore \quad d^3 = \frac{2584.23 \times 10^3 \times 32}{45 \times 2 \times z}$$

$$d = 66.37 \text{mm}$$

∴ Diameter of shaft is 66.37 mm