B. Tech.

SECOND SEMESTER EXAMINATION, 2008-09

Engg. Mechanics

Time : 3 Hours] **[Total Marks : 100**

Note : (1) This paper is in three sections. Section A carries 20 marks, Section B carries 30 marks and Section C carries 50 Marks.

(2) Attempt all questions. Marks are indicated against each question/part.

(3) Assume missing data suitably, if any

Section-A

Q. 1. You are required to answer all the parts : $2 \times 10 = 20$

Choose correct answer for the following parts :

(a) In order to determine the effect of a force acting on a body, we must know :

(i) its magnitude (ii) its direction

(ii) position or line of action (iv) all of these

Ans. (iv) all of these

(b) D'Alebert's principle is used for :

(i) determining the stresses in truss

(ii) reducing kinetic problem to equivalent statics problem

(iii) stability of floating bodies

(iv) solving kinetimatics problem

Ans. (ii) reducing kinetic problem to equivalent

Fill in the blanks for the following three parts :

You will be awarded full marks, if all the entries in a part the correct otherwise will be awarded zero.

(c) A truss is said to be rigid in nature when there is no .... on application of any external ....

(d) The centre of gravity of semicircle lies at a distance of .... whereas centre of gravity of a hemisphere lie at a distance of ...., both from its base measured along vertical axis.

(e) In a tensile test on mild steel specimen, the yield stress is .... than the ultimate stress and the breaking stress is .... than the ultimate stress if all the stresses are calculated on the basis of initial cross-sectional area of the speciman.

Ans. (c) Load, forces (d) $\frac{4r}{3\pi}, \frac{3r}{8}$ (e) less, less

Match the columns for the following three parts :

You will be awarded full marks, if all the matches in a part are correct otehrwise will be awarded zero.

(f) Match the following columns :

	Column-I		**Column-II**
(i)	Coplanar forces	(P)	Line of action of all forces lie in the same plane and pass through a common point
(ii)	Concurrent forces	(Q)	Line of action of all forces lie in the same plane
(iii)	Concurrent coplanar forces	(R)	Line of action of all forces lie along the same line
(iv)	Collinear forces	(S)	Line of action of all forces pass through a common point

Ans. (i) → Q (ii) → S (iii) → P (iv) → R

(g) Column-II gives the mass moment of inertia of various solids about the central axis. Match the following columns :

	Column-I		**Column-II**
(i)	Cylinder	(P)	$\frac{3mr^2}{2}$
(ii)	Sphere	(Q)	$\frac{3mr^2}{10}$
(iii)	Cone	(R)	$\frac{2mr^2}{5}$
(iv)	Tyin circular disk	(S)	$\frac{mr^2}{2}$

Ans. (g) (i) Cylinder ⟶ (S) $\frac{mr^2}{2}$ (ii) Sphere ⟶ (R) $\frac{2mr^2}{5}$

(iii) Cone ⟶ (Q) $\frac{3mr^2}{10}$ (iv) Thin circular disk ⟶ (P) $\frac{3mr^2}{2}$

(h) Column-II gives maximum bending moments for the following loads. Match the following columns :

	Column-I		**Column-II**
(i)	Cantilever with point load at free end	(P)	$\frac{Wl}{4}$

(ii) Cantilever with uniformly distributed load on the entires cantilever (Q) $\frac{Wl^2}{8}$

(iii) Simple supported beam with point load at mid span (R) Wl

(iv) Simple supported beam with uniformly distributed load on the entire beam (S) $\frac{Wl^2}{2}$

Ans. (i) Cantilever with point load at free end ⟶ (R) wl

(ii) Cantilever with uniformly distributed load on the entire ⟶ (S) $\frac{wl^2}{2}$

(iii) Simple supported beam with point load at mid span ⟶ (P) $\frac{wl}{4}$

(iv) Simple supported beam with uniformly distributed load on the entire beam ⟶ (Q) $\frac{wl^2}{8}$

Choose the correct answer for tne following two parts :

(i) Consider the following statemens :

(I) The magnitude of the moment does not change if the point of application of the force is transmitted along its line of action.

(II) The magnitude of the moment does not change if the moment centre is moved along a line parallel to the line of action of the force :

(i) I alone is correct (ii) I and II are correct

(iii) II alone is correct (iv) Neither I nor II correct

Ans. (i) I alone is correct

(j) Consider the following statements :

(I) In truss analysis the bars are connected at their ends by frictional hinges.

(II) In truss analysis the bars are assumed to be weightless.

(i) I alone is correct (ii) I and II are correct

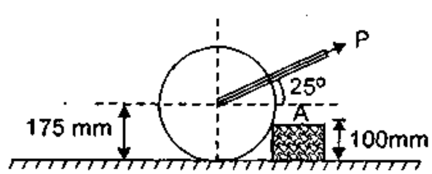
(iii) II alone is correct (iv) Neither I nor II is correct

Ans. (J) II alone is correct.

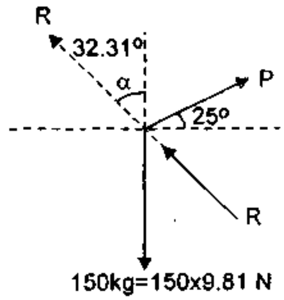
Section-B

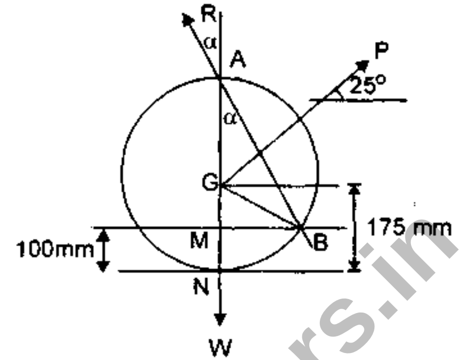
Q. 2. Answer the three parts of the following : **10 × 3 = 30**

Q. 2. (a) A roller shown in Fig. 1 is of mass 150 kg. What force *P* is necessary to start the roller over the block A ?



Ans. $BG = 175$ mm, $GM = 175 - 100 = 75$ mm, $MN = 100$ mm





$$BM = \sqrt{175^2 - 75^2} = 158.11 \text{ mm}$$

$$\tan\alpha = \frac{BM}{AM} = \frac{158.11}{(175+75)}, \quad \alpha = 32.31°C$$

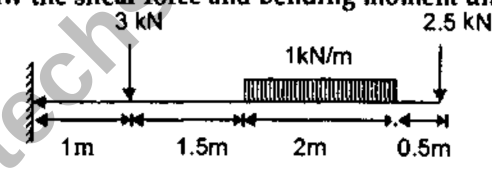
Now using Lamis theorem

$$\frac{P}{\sin(90 + 57.69)} = \frac{R}{\sin(25° + 90°)} = \frac{150\,g}{\sin(97.31)}$$

$\therefore$ $$P = 80.83\,gN = 792.96 \text{ N}$$

So to star the roller over the block 792.96 N force is necessary.

Q. 2. (b) Calculate the values of shear force and bending moments for the cantilever beam shown in Fig. 2, Also draw the shear force and bending moment diagrams.



Ans. For equilibrium $\Sigma F_V = 0$ $\quad\therefore\quad$ $R_A - 3 - 2 - 2.5 = 0$

$\therefore$ $$R_A = 7.5 \text{ km}$$

Moment about $A = M_A - 3 \times 1 - 2 \times 2.5 \times - 2.5 \times 5 = 0$

$\therefore$ $M_A = 22.5$ kN-m

Calculation for SF diagram (↑+)

S.F. between $BA = 7.5$ km, S.F. between $BC = 7\ 5 - 3 = 4.5$ km

S.F. between $CD = 4.5$ km, S.F. Between $ED = 4.5 - 2 = 2.5$ km

S.F. at $F = 0$

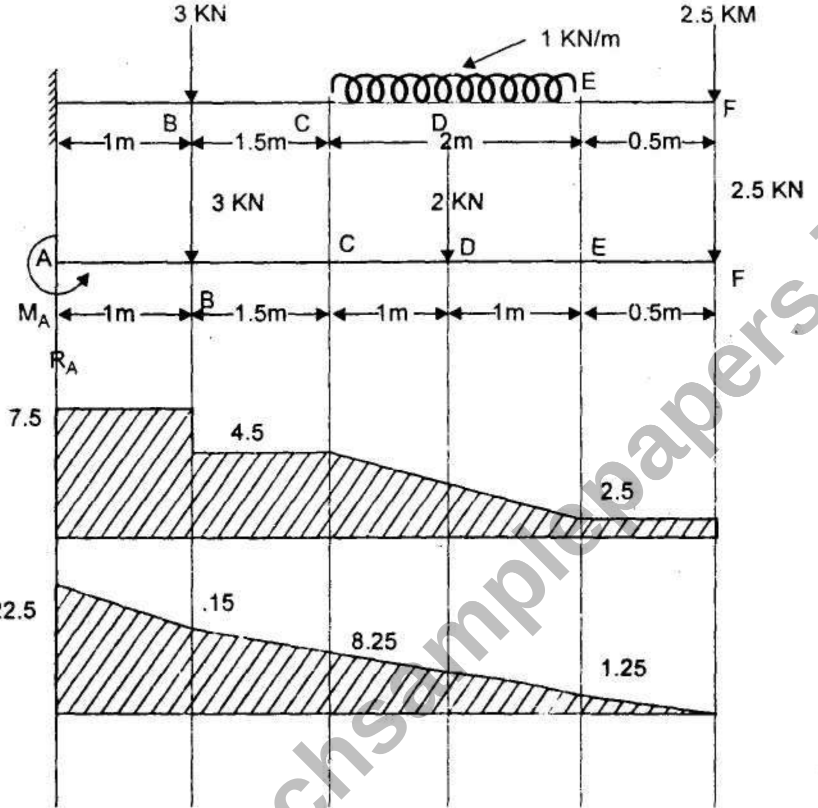
Calculate for B.M. diagram :

BM at point $A = 22.5$ kN-m

B.M. at point $B = 22.5 - 7.5 \times 1 = 15$ kN-m

B.M. at point $C = 22.5 - 7.5 \times 2.5 + 3 \times 1.5 = 8.25$ kN-m

B.M. at point $D = 22.5 - 7.5 \times 3.5 + 3 \times 2.5 = 3.75$ kN-m

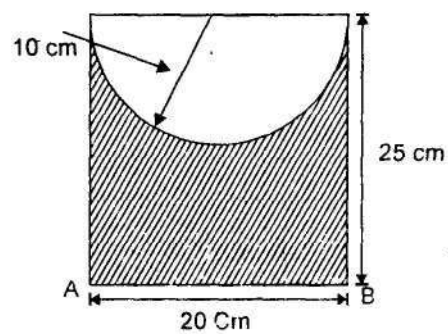


B.M. at point $E = 22.5 - 7.5 \times 4.5 + 3 \times 3.5 + 2 \times 1$

$= 1.25$ kN-m

B.M at point $F = 22.5 - 7.5 \times 5 + 3 \times 4 + 2 \times 1.5 = 0$

Q. 2. (c) Find the moment of inertia of the area shown shaded in Fig. 3 about edge AB.

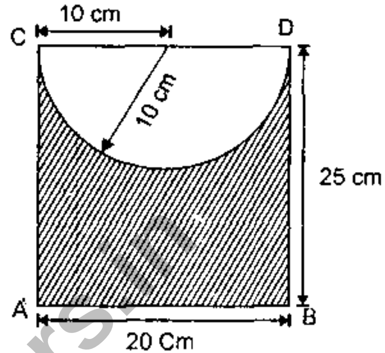


Ans. Moment of inertia of the shaded portion about AB

= MOI of reactangle $ABCD$ about AB — M.O.I. of semi circle of DC about AB

M.O.I. of rectangle $ABCD$ about $AB = \frac{bd^2}{3} = \frac{20 \times 25^3}{12}$

M.O.I of semicircle about $DC = \frac{1}{2} \times$ [MOI of a circle of radius 10 cm about a diameter]



$= 104{,}167 \text{ cm}^4 = \frac{1}{2} \times \left[\frac{\pi}{64} d^4\right] = \frac{1}{2} \times \frac{\pi}{64} [20]^4 = 3925 \text{ cm}^4$

Distance of C.G of semicircle from DC

$$= \frac{4r}{3\pi} = \frac{4 \times 10}{3\pi} = 4.24 \text{ cm}$$

Area of semicircle $(A) = \frac{\pi r^2}{2} = \frac{\pi \times 10^2}{2} = 157.1 \text{ cm}^2$

M.O.I. of semicircle about a line thorugh its C.G. parallel to CD

$= $ M.O.I. of semicircle about CD – Area × [Distance of C.G. of semicircle from $DC]^2$

$= 3925 - 157.1 \times 4.24^2$

$= 3925 - 2824.28 = 1100.72 \text{ cm}^4$

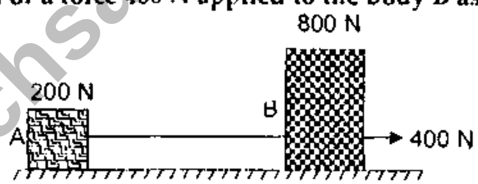
Distance of C.G. of semicircle from $AB = 25 - 4.24 = 20.76$ cm

M.O.I. of semicircle about $AB = 1100.72 + 157.1 \times 20.76^2$

$= 68807.30 \text{ cm}^4$

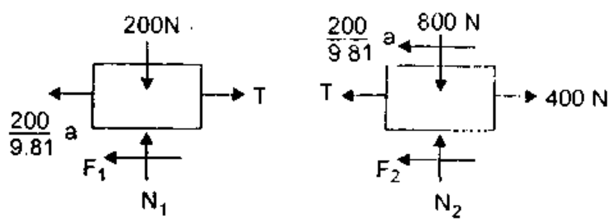
$\therefore$ M.O.I. of shaded portion about $AB = 104167 - 68807.30 = 35359.7 \text{ cm}^4$

Q. 2. (d) Two bodies A and B are connected by a thread and move along a rough horizontal plane (μ = 0.3) under the action of a force 400 N applied to the body *B* as shwon in Fig.



Determine the acceleration of the two bodies and the tension in the thread, using D' Alembert's principle.

Ans. Free body diagram of 200 N and 800 N blocks along with inertia forces are shown in figure in which a is acceelration of the system.



Consider the dyanamic eqiulibrium of 200 N weight $\Sigma V = 0, V_1 = 200$ N

From law of function $F_1 = \mu N_1 = 0.3 \times 200 = 60$ N

$\Sigma H = 0$

$$T - F_1 - \frac{200}{9.81} a = 0$$

$$\therefore \quad T - \frac{200}{9.81} a = 60 \quad \ldots(1)$$

as $F_1 = 60$ N

Consider 800 N body, $\Sigma V = 0$, $N_2 = 800$ N

From law of function $F_2 = \mu N_2 = 0.3 \times 800 = 240$ N

$\Sigma H = 0$

$$\therefore \quad -T - \frac{800}{9.81} a - F_2 + 400 = 0$$

$$\text{or} \quad T + \frac{800}{9.81} a = 160 \qquad \ldots(2)$$

as $F_2 = 240$ N

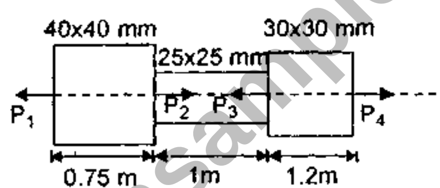
By solving Eq. (1) and (2) we get $\left(\frac{200}{9.81} + \frac{800}{9.81} \right) a = 160 - 60$

$\therefore \quad a = 0.981 \text{ m/sec}^2$

Substituting in Eq. (2) we get $T = 160 - \frac{800}{9.81} \times 0.981$

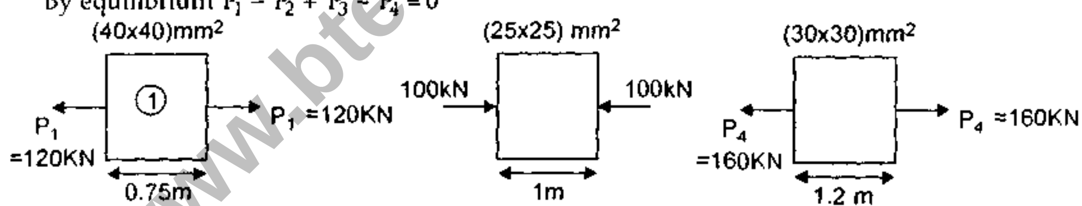
$$T = 80 \text{ N}$$

Q. 2 (e) A member is subjected to point loads P_1, P_2, P_3 and P_4 as shown in Fig. Calculate the force P_3 necessary for equilibrium if $P_1 = 120$ kN, $P_2 = 220$ kN and $P_4 = 160$ kN. Determine also the change in length of the member. Take $E = 2 \times 10^5$ N/m².



Ans. $P_1 = 120$ kN $E = 2 \times 10^5$ N/mm², $P_2 = 220$ kN, $P_4 = 160$ kN

By equilibrium $P_1 - P_2 + P_3 - P_4 = 0$



$\therefore \quad P_3 = P_4 + P_2 - P_1 = 160 + 220 - 120 = 260$ kN

$$\delta L_1 = \frac{P_1 L_1}{A_1 E} = \frac{120 \times 0.75 \times 10^6}{40 \times 40 \times 2 \times 10^5} = 0.28 \text{ mm } (+\text{ve})$$

$$\delta L_2 = \frac{P_2 L_2}{A_2 E} = \frac{100 \times 1 \times 10^3 \times 10^3}{25 \times 25 \times 2 \times 10^5} = 0.8 \text{ mm } (-\text{ve})$$

$$\delta L_3 = \frac{P_3 L_3}{A_3 E} = \frac{160 \times 10^3 \times 1.2 \times 10^3}{30 \times 30 \times 2 \times 10^5} = 1.067 \ (+\text{ve})$$

So charge is length of bar $= \delta L_1 + \delta L_2 + \delta L_3 = 0.28 - 0.8 + 1.067 = 0.547$ mm

Section-C

Q. 3. Answer any two parts of the following : **5 × 2 = 10**

(a) Explain the following ;

(i) Laws of static friction **(ii) Limiting angle of friction.**

Ans. (i) Laws of static friction :

(1) Friction force is always acting in opposite direction in which the body tends to move.

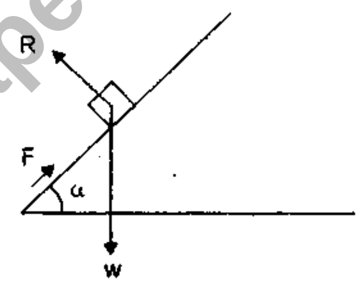
(2) Friction force is directly proportional to normal reaction $F_{max} = \mu_s R$.

(3) The Friction force is equal to the net force acting in the direction of tendency of motion.

(4) The frictional force is independent of the surface area in contact.

(5) The frictional force depends upon the nature or rougness of the surface.

(ii) Limiting angle of friction : Consider a body of weight w resting on an inclined plane as shown in figure, we know that the body is equilibrium under the action of the following forces



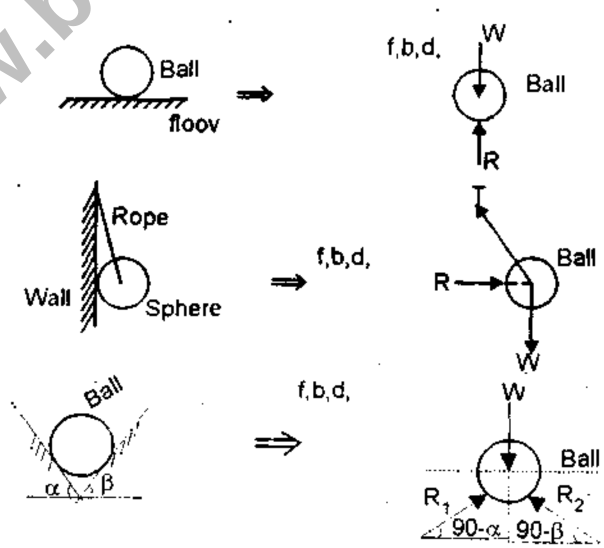
1. Weight (w) of the body acting vertically downwards
2. Friction force (F) acting upwards along the plane.
3. Normal reaction (R) acting at right angles to the plane

Let the angle of inclination (α) be gradually increased till the body just starts sliding down the plane. This angle of icnline d plane, at which a body just begins to slide down the plane is called the angle of friction. This is also equal to the angle, which is normal reaction makes with the vertical.

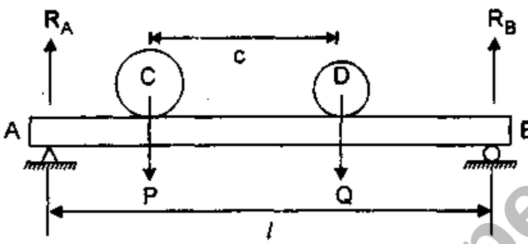
Q. 3. (b) What is a free body diagram ? Explain with suitable example.

Ans. Free Body Diagram : f.b.d. shows a body isolated from its surroundings and all external actions acting on it. In other word, in a free body diagram, all the supports (like walls, floors, hinger etc.) are removed and replaced by the reactions which these supports exert on the body.

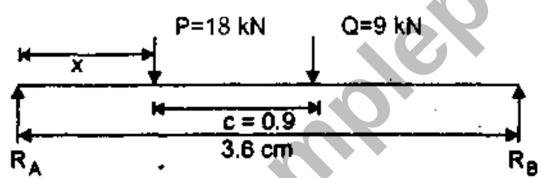
Example :



Q. 3. (c) Two rollers *C* and *D* produce vertical forces *P* and *Q* on the horizontal beam *AB*, as shown in Fig. Determine the distance *x* of the load *P* form the support. A if the reaction at A is twice as great as the reaction at *B*. The weight of the beam is to be neglected. Given : $P = 18$ kN, $Q = 9$ kN, $l = 3.6$ m, $c = 0.9$ m



Ans. $R_A = 2R_B$ ...(1)



moment about *A* is $R_A \times 0 + 18 \times x + 9 \times (0.9 + x) - R_B \times 3.6 = 0$ ...(2)

For vertical components of forces $\Sigma F_V = R_A + R_B - 18 - 9 = 0$

$\therefore \quad R_A + R_B = 18 + 9 = 27$ kN

$\therefore$ By substitute of Eq. (1)

$\therefore \; 3R_B = 27$ kN, $\quad R_B = 9$ kN, $\quad R_A = 18$ kN

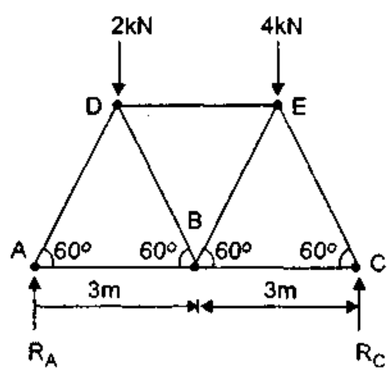
Now Eq. (2)

$\Rightarrow \; 18x + 9(0.9 + x) = 3.6R_B \quad \therefore \quad 18x + 9(0.9 + x) = 3.6 \times 9, \quad x = 0.9$ m

so distance *x* is 0.9 m.

Q. 4. Answer any one of the following : 10

Q. 4. (a) Find the axial forces in all members of a truss as shown in Fig.



Ans. Consider the equilibrium of the entire frame

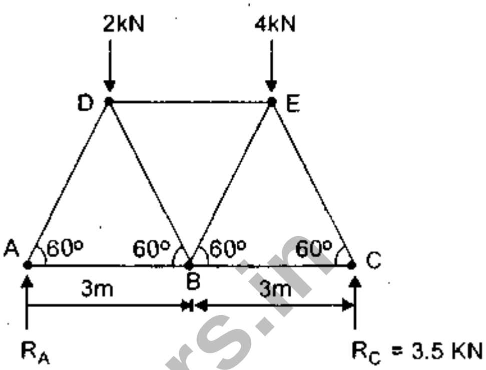
$\Sigma M_A = 0$

$$R_A \times 0 + 2 \times 1.5 + 4 \times 4.5 - R_C \times 6 = 0$$

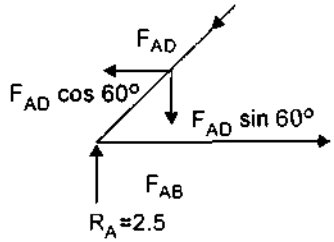
$$R_C = 3.5 \text{ km}$$

$$\Sigma F_V = 0, R_A + R_C = -2 - 4 = 0$$

$$R_A = 2.5 \text{ kN}$$



∴ Joint A :



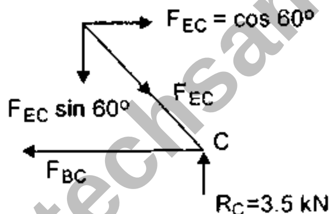
$(\overset{+}{\rightarrow})$

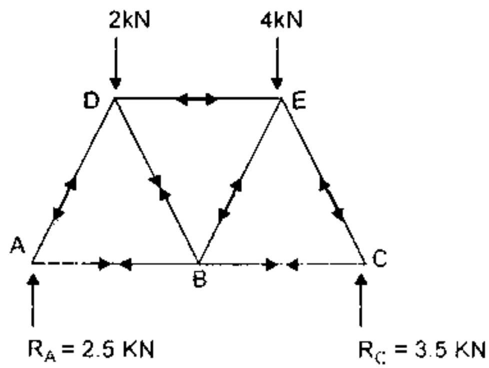
$$\Sigma F_H = F_{AB} - F_{AD} \cos 60° = 0$$

$$(+\uparrow)\Sigma F_V = R_A - F_{AD} \sin 60 = 0$$

∴ $$F_{AD} = \frac{R_A}{\sin 60} = \frac{2.5}{\sin 60°} = 2.89 \text{ kN}$$

Joint C :





$F_{AB} = 1.44$ kN, $\quad (\overset{+}{\rightarrow}) \Sigma F_H = 0$

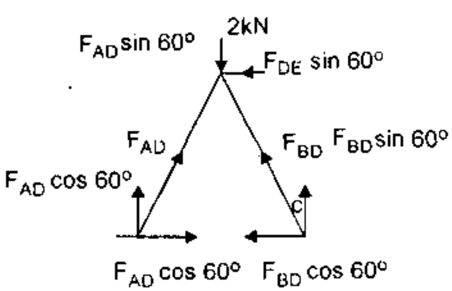
$$F_{EC} \cos 60° - F_{BC} = 0$$

$$(+\uparrow)\Sigma F_V = 0 \; F_{EC} \sin 60° = 3.5$$

∴ $$F_{EC} = 4.04 \text{ kN}$$

$$F_{BC} = 2.02 \text{ kN}$$

For the joint D :

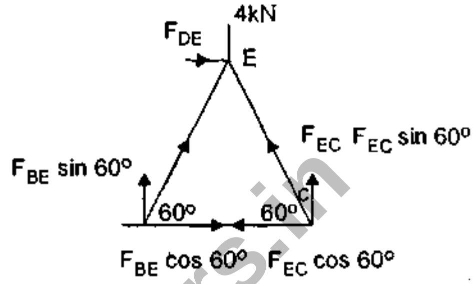


$\Sigma F_V = F_{AD} \sin 60° + F_{BD} \sin 60° - 2 = 0$

$\Sigma F_H = F_{AD} \cos 60° - F_{BD} \cos 60° - F_{DE} = 0$

$F_{BD} = \mathbf{0.51\ kN}$

For the joint E :



$\Sigma F_V = 0, F_{EC} \sin 60° + F_{BE} \sin 60° = 4$

$\Sigma F_H = 0, F_{EC} \cos 60° = F_{BC} \cos 60° - F_{DE}$

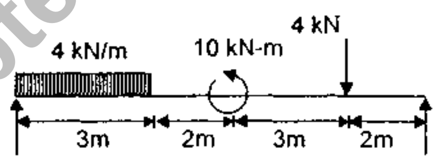
$F_{BE} \sin 60° = 4 - F_{EC} \sin 60°$

$F_{BE} = .493$ kN

$F_{DE} = 1.778$ kN

Name of force	Magnitude	Nature
F_{AD}	2.89 kN	(C)
F_{AB}	1.44 kN	(T)
F_{BC}	2.02 kN	(T)
F_{EC}	4.04 kN	(C)
F_{BD}	0.51 kN	(T)
F_{BE}	.493 kN	(C)
F_{DE}	1.778 kN	(C)

Q. 4. (b) Draw the shear force and bending moment diagram for the beam loaded as shown in Fig.



Ans. By equilibrium condition $(+\uparrow)\ \Sigma F_V = 0$, So $R_A + R_E - 12 - 4 = 0$

$\therefore \quad R_A + R_E = 16$

$\Sigma m_A = R_A \times 0 - 12 \times 1.5 + 10 - 4 \times 8 + R_E \times 10 = 0$

$\therefore\ R_E = 4$ kN $\qquad \therefore \quad R_A = 12$ kN

$\therefore$ **S.F. Calculation**

$(+\uparrow)$ S.F. between $AF = 12$ kN

S.F. between $BF = 12 - 12 = 0$

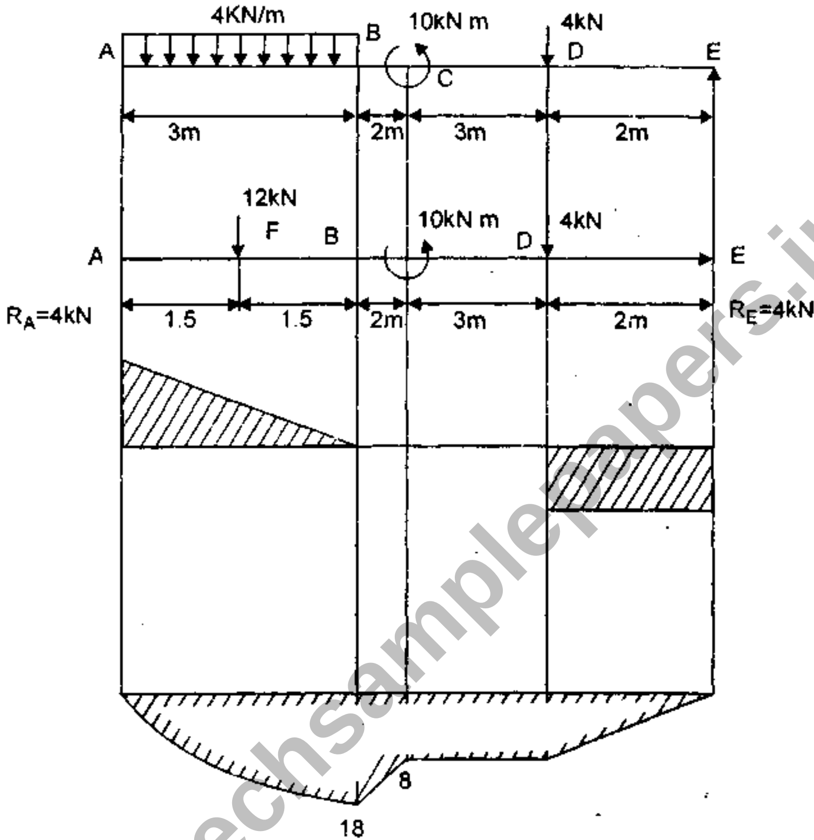
S.F. between $BD = 0$

S.F. between $DE = -4$ kN

S.F. at $E = -4 + 4 = 0$

B.M. Calculation :

B.M. at $A = 0$



B.M. at $F = -12 \times 1.5 = -18$ kN-m

B.M. at $B = -12 \times 3 + 12 \times 1.5 = -18$ kN·m

B.M. at $C = -12 \times 5 + 12 \times 3.5 + 10 = -8$ kN-m

B.M. at $D = -12 \times 8 + 12 \times 6.5 + 10 = -8$ kn-m

B.M. at $E = -12 \times 10 + 12 \times 8.5 + 10 + 4 \times 2 = 0$

Q. 5. Answer any two parts of the following : **$5 \times 2 = 10$**

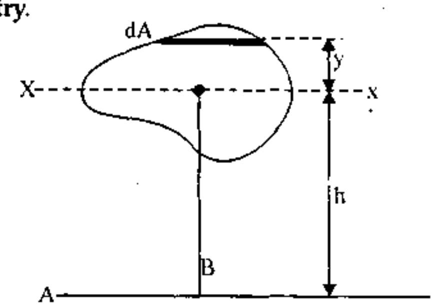
(a) Explain any two of the following :

(i) Parallel axis theorem

(ii) Product of inertia of an area about its axis of symmetry.

(iii) Centre of gravity.

Ans. (i) Theorem of Parallel Axis



In the figure shown below,

If A = Area of plane figure

I_G = Moment of inertia of the given area about

$$\text{C.G.} = I_{XX}$$

I_{AB} = Moment of inertia of the given area about AB axis.

h = Distance between c.g. of area (*i.e.*, XX axis) and axis AB parallel to XX axis.

Then, $$I_{AB} = I_G + Ah^2$$

Proof—Let XX-axis is the axis in the plane of area A, passing through the C.G.of the area.

Let AB-axis is the axis in the plane of area A, parallel to the axis XX.

Consider, an elementary area dA at a distance y from XX-axis. Taking moment of inertia of this dA about XX-axis, we get $dA.y^2$.

$\therefore$ The moment of inertia of whole area about XX-axis.

$$I_{XX} = I_G = \Sigma dA.y^2$$

Moment of inertia of dA about axis AB is $dA.(y + h)^2$

$\therefore$ The moment of inertia of whole area about AB axis = I_{AB}

$$I_{AB} = \Sigma dA\,(y + h)^2$$
$$= \Sigma dA\,(y^2 + h^2 + 2hy)$$
$$= \Sigma dA\,.\,y^2 + \Sigma dA\,.\,h^2 + \Sigma dA\,.\,2hy = I_G + Ah^2 + 0$$

($\because 2h\,\Sigma dAy = 2h \times 0 = 0$) As, $\Sigma dA = A$ and y is the distance between the c.g. of the area (*i.e.*, G) and the axis XX.

Therefore, $$y = 0$$

Hence, $$I_{AB} = I_G + Ah^2$$

This theorem is very useful in transferring moment of inertia from centroidal axis to a desired axis parallel to the centroid axis.

Cor. If K_{xx} and K_{AB} are radius of gyration about axis XX and AB respectively, then

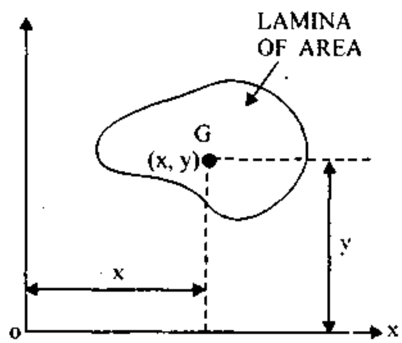
$$AK_{AB}^2 = A\,K_{XX}^2 + Ah^2 \qquad (\because\ I = AK^2)$$

$\therefore$ $$K_{AB}^2 = K_{XX}^2 + h^2$$

If we consider a solid body and if M = mass of solid body

Then, $$I_{AB} = I_G + Mh^2$$

(ii) Moment of inertia : It is the product of area or mass with the square of the perpendicular distance of its c.g. from axis of reference. It is denoted by I.



In above Fig. OX and OY are two axes are reference. A thin lamina of area 'A' having its c.g. at G is shown in Fig. The co-ordinates of G are (x, y) and whole of the area 'A' is concentrate at G.

Moment of this area A, about $OY = A\,.\,x$

This is known as first moment of area OY.

Now, taking moment of this "moment of area" about OY, we get

$$I_{YY} = (A.x) \cdot x = Ax^2$$

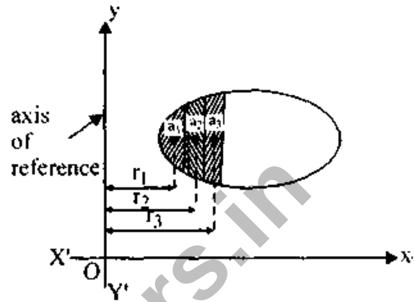
Where, I_{YY} = second moment of area about Y axis.

= Moment of Inertia about Y-axis.

Similarly by taking second moment of area about OX, we get

$$I_{XX} = (A \cdot y) \cdot y = Ay^2$$

Where, I_{XX} = Moment of Inertia about X-axis.



If, instead of area, mass (m) of the body is taken into consideration then the second moment is known as second moment of mass. This second moment of mass is known as "mass moment of inertia."

Hence, moment of inertia (when mass is taken into consideration) about $OY = mx^2$ and about the axis $OX = my^2$

In above Fig., the plane area is sptitted into a number of small areas $a_1, a_2, a_3, ..$ etc. having their individual c.g. at a distance $r_1, r_2, r_3, ...$ etc. from axis of reference. Then the moment of inertia of the plane area about the given axis is given by

$$I = a_1r_1^2 + a_2r_2^2 + a_3r_3^2 + ...$$

$$I = \Sigma ar^2$$

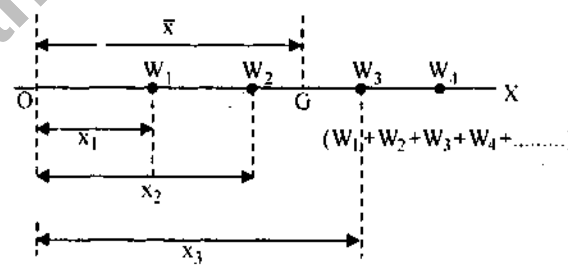
(iii) Centre of gravity : Centre of gravity of a body is the point through which, the whole weight of body acts.

Following are the few points in connection with the centre of gravity :

(a) A body is having only one centre of gravity.

(b) The centre of gravity of the body does not alter even if its positions changed.

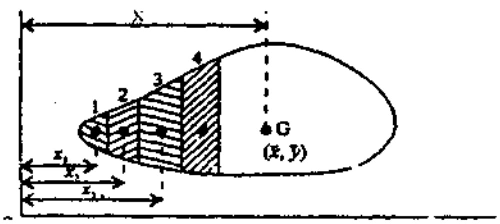
(c) It is an imaginary point which may occur inside or outside the body.



Consider the weight $w_1, w_2, w_3, ...$. These are placed on axis OX. The distances from O are x_1, x_2, x_3. Let G is the c.g. of all these weights and its distance O is $\bar{x}$. Taking moments about O, we have,

$$(w_1 + w_2 + w_3 + ...)\,\bar{x} = w_1x_1 + w_2x_2 + w_3x_3 + ...$$

$$\therefore\ \bar{x} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + ...}{w_1 + w_2 + w_3 + ...} = \frac{\Sigma wx}{\Sigma w} = \frac{\Sigma wx}{W}$$



If a plane area A is divided into strips 1, 2, 3 etc. and if these strip areas $a_1, a_2, a_3, ...$, each hving its centre of gravity (x_1, y_1), (x_2, y_2), (x_3, y_3), ..., then the centroid of whole area $(a_1 + a_2 + a_3 + ...)$ is at G having co-ordinate $(\bar{x}, \bar{y})$

By taking moments about OY, we have

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + ...}{a_1 + a_2 + a_3 + ...} = \frac{\Sigma ax}{\Sigma a}$$

By taking moments about OX, we have

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + ...}{a_1 + a_2 + a_3 + ...} = \frac{\Sigma ay}{\Sigma a}$$

we have seen

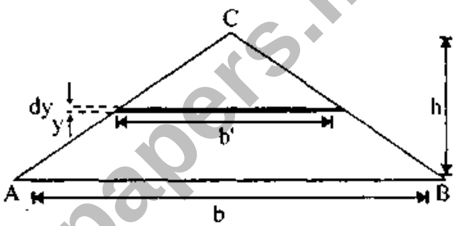
$$\bar{x} = \frac{\Sigma wx}{\Sigma w}$$

If the body is made of the same material, the density (ρ) of each particle is to be same. Then $w_1 = v_1$, $w_2 = v_2.\rho$ etc, where $v_1, v_2, ...$ are the volume of the particles. Hence, we get

$$\bar{x} = \frac{\Sigma v.\rho .x}{\Sigma v.\rho} = \frac{\Sigma vx}{\Sigma v}$$

Example.

The C.G. of a triangle



ABC is a triangular whose base is b and height is h, as shown in the figure. Here, $dA = b'.dy$ = area of element.

$$A = \int_0^h b'\,dy \qquad \frac{b}{h} = \frac{b'}{h-y}$$

$$= \int_0^h \frac{b}{h}(h-y)\,dy \qquad \therefore \qquad b' = \frac{b}{h}(h-y)$$

$$= -\frac{b}{h}\left[\frac{(h-y)^2}{2}\right]_{y=0}^{y=h} = -\frac{b}{2h}[0-h^2]^{y=0}$$

$$A = +\frac{bh^2}{2h} = \frac{1}{2}bh$$

Let $\bar{y}$ be the distance of C.G. of triangle from base.

Then, $$\bar{y} = \frac{\int dA.y}{\int dA} = \frac{\int_0^h \frac{b}{h}(h-g)\,dy\,.\,y}{\frac{1}{2}bh} = \frac{\frac{b}{h}\int_0^h (h-y)\,y\,.\,dy}{\frac{1}{2}bh}$$

$$= \frac{\frac{b}{h}\int_0^h (hy - y^2)\,dy}{\frac{1}{2}bh}$$

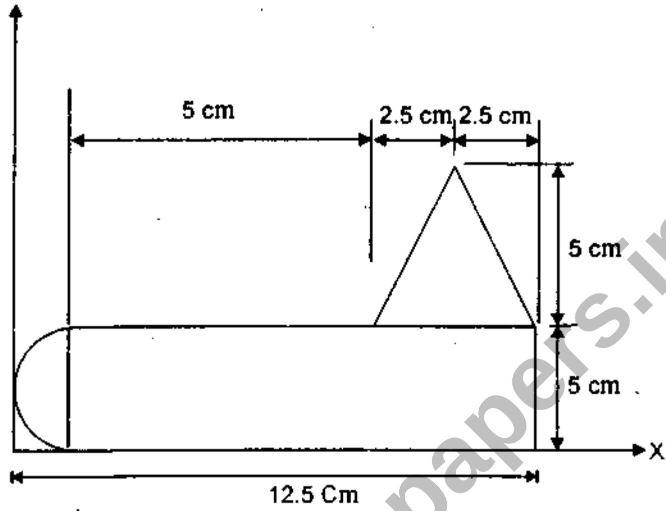
$$= \frac{2b}{h.bh}\left[\int_0^h hy.dy - \int_0^h y^2\,dx\right] = \frac{2}{h^2}\left[h\left(\frac{y^2}{2}\right)_0^h - \left(\frac{y^3}{3}\right)_0^h\right]$$

$$= \frac{2}{h^2}\left[\frac{h^3}{2} - \frac{h^3}{3}\right] = \frac{2}{h^2}h^3\left[\frac{3}{6} - \frac{2}{6}\right] = 2h \times \frac{1}{6} = \frac{h}{3}$$

Centre of Mass : It is the point where whole mass of the body is concentered.

Centroid or Centre of Area : It is the centre of a uniform plane area. If the density of the material is same and thickness of area is also same, then centre of area is called as centroid.

Q. 5. (b) Find the centroid of Fig.



Ans. Area of semicircle $\frac{1}{2}\left(\frac{\pi}{4}d^2\right) = \frac{1}{2}\left[\frac{\pi}{5}(5)^2\right] = A_1 = 39.27 \text{ cm}^2$

Area of rectangle $A_2 = 10 \times 5 = 50 \text{ cm}^2$

Area of triangle $A_3 = \frac{1}{2} \times 5 \times 5 = 12.5 \text{ cm}^2$

The coordinates of centroides of these three simple figures are

$$x_1 = 2.5 - \frac{4R}{3\pi} = 2.5 - \frac{4 \times 2.5}{3\pi} = 1.44 \text{ cm}$$

$$y_1 = 2.5 \text{ cm}$$

$$x_2 = 2.5 + \frac{10}{2} = 7.5 \text{ cm}$$

$$y_2 = 2.5 \text{ cm}$$

$$x_3 = 2.5 + 5 + 2.5 = 10 \text{ cm}$$

$$y_3 = 5 + \frac{h}{3} = 5 + \frac{5}{3} = 6.67$$

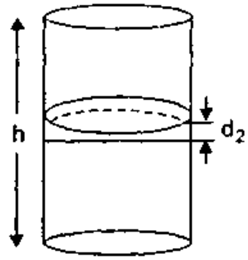
$$\therefore \quad \bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \frac{39.27 \times 1.44 + 50 \times 7.5 + 12.5 \times 10}{39.27 + 50 + 12.5} = 5.47 \text{ cm}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = \frac{39.27 \times 2.5 + 50 \times 2.5 + 12.5 \times 6.67}{39.27 + 50 + 12.5} = 3.01 \text{ cm}$$

Q. 5. (c) Derive an expression of mass moment of inertia of a cylinder about the longitudinal axis.

Ans. Consider the cylinder to be made up of series of thin disks of height dz as shown in figure.

The M.I. of such a disk $= \frac{\text{mass} \times (\text{radius})^2}{2} = \frac{1}{2}\frac{(m\,dz)}{h}r^3$



For entire cylinder $I = \int_0^h \frac{m\,dz}{h} r^2 = \frac{1}{2} Mr^2$

Q. 6. Answer any two parts of the following : $5 \times 2 = 10$

Q. 6 (a) What do you understand by the term kinematics ? Explain different types of plane motion of rigid bodies with suitable example.

Ans. When a particle moves in a space it describes a curve called path. This path can be straight or curved.

(i) Rectilinear motion–When a particle moves along a path which is a straight line, it is called rectilinear motion.

(ii) Curvilinear motion : when a particle moves along a curved path it is called curvilinear motion. If the curved path lines in a plane it is called plane curvilinear motion.

Dynamic is the part of mechanics that deals with the analysis of bodies in motion.

Kinematics is the study of the relationships between displacement, velocity acceleration and time of a given motion, without considering the forces that cause the motion.

Kinetics is the study of the relationships between the forces acting on a body, the mass of the body and the motion of the body. So kinetics use to predict the motion caused by a given force or to determine the forces require to produce a prescribe motion.

Q. 6. (b) A wheel rotates for 5 seconds with constant angular acceleration and describes 100 radians during this time. It then rotates with a constant angular velocity and during the next 5 seconds describes 80 radians. Find the initial angular velocity and the angular acceleration.

Ans. $t_1 = 5$ sec, $\alpha, \theta = 100$ rad, $\omega_0 \rightarrow$ initial angular velocity

$t_2 = 5$ sec, $\omega, \theta = 80$ rad, $\omega \rightarrow$ final angular velocity

$\theta = \omega_0 t_1 + \frac{1}{2}\alpha t^2,$ $\qquad 100 = \omega_0 \times 5 + \frac{1}{2}\alpha(5^2)$

$$100 = 5\omega_0 + \frac{25\alpha}{2} \qquad \ldots(1)$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta \qquad \ldots(2)$$

For 2nd case $\theta = \cos \omega t$

$\therefore \; 80 = \omega \times 5 \qquad \therefore \qquad \omega = 60$ rad/sec

$$16^2 - \omega_0^2 = 2 \times \alpha \times 100 \qquad \ldots(3)$$

$$= 2 \times \frac{[200 - 10\omega_0]}{25} \times 100$$

$$16^2 - \omega_0^2 = 1600 - 80\omega_0$$

$$\omega_0^2 - 80\omega_0 + 1344 = 0, \qquad \omega_0 = 24 \text{ rad/sec}$$

$$\therefore \quad \alpha = \frac{16^2 - 29^2}{2 \times 100} = 1.6 \text{ rad/sec}^2$$

$\therefore$ Initial angular velocity 24 rad/sec

Angular acceleration 1.6 rad/sec.

Q. 6. (c) What is energy ? Explain the various forms of mechanical energies.

Ans. Energy is defined as capacity of to do work. Mechanical energy which is classified into potential energy and kinetic energy. Potential energy is the capacity to do work due to position of the body. A body of weight W held at a height h posseses an energy Wh Kinetic energy is the capacity to do work due to motion of the body.

Energy is defined as capacity to do work. The energy can exist in numerous forms such as thermal, mechanical, kinetic, potential and nuclear energy. The sum of all these constituent of energy is called totalenergy or energy E of a system. The energy of a system on unit mass basis is denoted as e and is defined as $e = \frac{E}{m}$.

The energy of a system pos... as a result of motion relative of some reference is called kinetic energy. When all parts mass m of a system moves with same velocity V, the kinetic energy is expressed as

$$K.E. = \frac{mV^2}{2} \quad \text{or} \quad \text{unit mass basis } K_e = \frac{V^2}{2}$$

The energy that a system posses as a result of its elevation in a gravitational field is called potential energy p.E is expressed as

$$PE = mgz$$

$Pe = gZ$ on unit mass basis

Where g is the acceleration due to gravity and Z is the elevation of the system relative to some outside referrence.

Q. 7. Answer any one of the following : **10**

Q. 7. (a) What do you understand by the term neutral axis and neutral surface ? A steel beam of hollow square section of 60 mm outer side and 50 mm inner side is simply suported on a span of 4 meters. Find the maximum concentrated load the beam can carry at the middle of the spain if the bending stress is not to exceed 120 N/mm^2.

Ans. In a beam, a layer or surface which does not charge its original length even after bending is called as neutral layer as neutral surface. The bending stress is always zero at neutral surface. Neutral axis is the line of interseciton of the neutral laryer with any normal section of the beam. It is proved that the neutral axis of the beam passes thorugh the centroid of the section.

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{50 \times 60^3}{12} - \frac{60 \times 50^3}{12} = 559166.67 \text{ mm}^4$$

$$y = \frac{60}{2} = 30 \text{ mm}$$

$M = \frac{Wl}{4}$ where W = is the concentric load

$l = 4 \times 10^3$ mm $\quad \frac{M}{I} = \frac{\sigma}{y} \quad \therefore \quad \frac{W \times 4 \times 10^3}{4 \times 559166.67} = \frac{120}{30}$

$\therefore W = 2236.67$ N

So maximum concentratrated load the beam carry is 2236.67 N

Q. 7. (*b*) State the assumption made in the thoery of pure torsion.

In a tensile test, a test piece 25 mm in diameter, 200 nm guage length stretched 0.0975 mm under a pull of 50,000 N. In a torsion test, the same rod twisted 0.025 radian over a length of 200 mm, when a torque of 400 Nm was applied. Evaluate the Poisson's ratio and the three elastic moduli for the material.

Ans. Following assumptions are made in theory of pure torsion

(i) The material of the shaft is uniform through out.

(ii) The twist along the shaft is uniform.

(iii) The normal cross section of the shaft which was straight before twist remain plane and circular after twist.

(iv) All diameter of the normal cross section remains unchange, *i.e.*, straight

Tensile Test : $d = 25$ mm, $L = 200$ mm, $\Delta L = 0.0975$ mm $P = 50{,}000$ N

Tortion Test : $\theta = 0.025$ radians $L = 200$ mm, $T = 400$ N-m

$$\text{Poisson ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\frac{T}{I_p} = \frac{C\theta}{l} = \frac{f_s}{R}$$

$$I_p = \frac{\pi d^4}{32} = \frac{\pi \times 25^4}{32} = 38349.52 \text{ mm}^4$$

$$C = \frac{Tl}{I_p\theta} = \frac{400 \times 10^3 \times 200}{38349.5 \times 0.025} = 8344.30 \text{ N/mm}^2 \text{ (modulus of origidity)}$$

$$E = \frac{\frac{P}{A}}{\frac{\Delta L}{L}} = \frac{\frac{50000}{\frac{Z}{4} \times (25)^2}}{\frac{0.0975}{200}} = 208941.874 \text{ N/mm}^2 \text{ (Modulus of elasticity)}$$

$$E = 2C(1 + \nu)\nu \rightarrow \text{Poisson ratio}$$

$$208941.874 = 2 \times 8344.30 (1 + \nu)$$

$$\therefore \quad \nu = 11.52 \text{ (Poisson ratio)}$$

$$E = 3K(1 - 2\nu) \text{ where } K \text{ is bulk modulus}$$

$$208941.874 = 3K(1 - 2 \times 11.52)$$

$$K = 3160.04 \text{ N/mm}^2$$