# B. Tech.

## SECOND SEMESTER EXAMINATION 2009-10

## ENGINEERING MECHANICS

**Time : 3 Hours** **Total Marks : 100**

**Note :** (i) This paper is in **three** sections. Section A carries 20 marks, Section B carries 30 marks and Section C carries 50 marks.

(ii) Attempt all questions. Marks are idicated against each question part.

(iii) Assume missing data suitably, if any.

### Section-A

**Q. 1. You are required to answer all the parts :**

**Choose correct answer for the following parts.**

**(a) The necessary and sufficient condition for a system of coplanar forces to be in equilibrium.**

(i) $\Sigma F_x = 0$ (ii) $\Sigma F_x = \Sigma F_y = 0$

(iii) $\Sigma M_0 = 0$ (iv) $\Sigma F_x = \Sigma F_y = \Sigma M_0 = 0$

**(b) The bending equation is :**

(i) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ (ii) $\frac{M}{y} = \frac{\sigma}{I} = \frac{E}{R}$

(iii) $\frac{M}{y} = \frac{\sigma}{R} = \frac{E}{I}$ (iv) $\frac{M}{I} = \frac{\sigma}{R} = \frac{E}{y}$

**(c) The principle of conservation of energy can't be applied in the following situation :**

(i) body sliding down on a rough inclined plane.

(ii) simple pendulum

(iii) a particle executing SHM

(iv) a particle moving in a gravitational field

**(d) In UDL loading (ω N/ m), the maximum bending moment in case of simple supported beam is given as :**

(i) $\omega L$ (ii) $\frac{\omega L^2}{2}$

(iii) $\frac{\omega L^2}{4}$ (iv) $\frac{\omega L^2}{8}$

Ans. 1. (a) (i) $\Sigma F_x = 0$ (b) (ii) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ (c) (d) (iv) $\frac{\omega L^2}{8}$

**Fill in the blanks for the following parts :**

**You will be awarded full marks, if all the entries in a part are correct otherwise will be awarded zero.**

**(e) The algebric sum of the moments of two ................ forces with respect to any moment centre in their plane of action is equal to the moment of their .................. with respect to the same centre.**

**(f) In a cantilever beam carrying a concentrated load at the free end, the bending moment will be zero at .................. and maximum at .................. .**

**(g) The angular velocity (rad/sec) of a body rotating at N rpm is .................. and the linear velocity of n body rotating at ω rad/sec along a circular path of radius r is .................. .**

**(h) In truss analysis, all forces acting on truss are applied at the .................. only nnd also lie in the ...................... of truss.**

**Ans.** (e) **Coplaner** forces with respect to any moment centre .................. to the moment of their **resultant force** with respect to the same centre.

(f) The bending moment will zero at **starting point** and **maximum at ending** or where it is hinged.

(g) $\frac{2zN}{60}$ rad/sec, $r\frac{d\theta}{dt}$

(h) Joint only and also lie in the member of the truss.

**Match the columns for the following parts :**

**You will be awarded full marks, if all the matches in a part are correct otherwise will be awarded zero.**

**(i) Match the following columns. Column II shows the moment of inertia about a centrodal axis :**

| | Column I | | Column II |
|---|---|---|---|
| (i) | Triangle | (P) | $0.11R^4$ |
| (ii) | Circle | (Q) | $\frac{\pi R^4}{4}$ |
| (iii) | Semicircle | (R) | $\frac{bh^3}{12}$ |
| (iv) | Rectangle | (S) | $\frac{bh^3}{36}$ |

**(j) Match the following columns :**

| | Column I | | Column II |
|---|---|---|---|
| (i) | Curvilinear motion | (P) | Neither pure rotation nor pure translation |
| (ii) | Rectilinear motion | (Q) | Pure rotary motion |
| (iii) | General plane motion | (R) | Motion of particles remains parallel and straight |
| (iv) | Instantaneous motion | (S) | Motion of particles remains parallel and in curve |

**Ans.** (i) (i) → (S) (ii) → (Q) (iii) → (P) (iv) → (R)

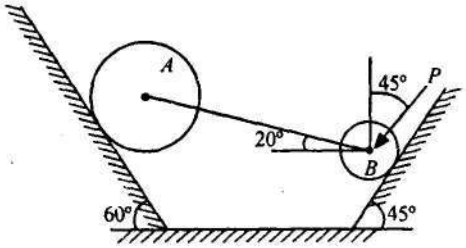
(j) (i) → (S) (ii) → (Q) (iii) → (R) (iv) → (P)

## Section-B

**Q. 2. Answer any three parts of the following :**

**(a) Two cylinders A and B weighing 4 kN and 3 kN, respectively, rest on smooth inclined plane as shown in Figure 1. They are connected by a bar of negligible weight hinged to each cylinder at its**

**geometric centre by smooth pins. Find the force P to be applied to the smaller cylinder at 45° to the vertical to hold the system in the given position.**




**Ans.**

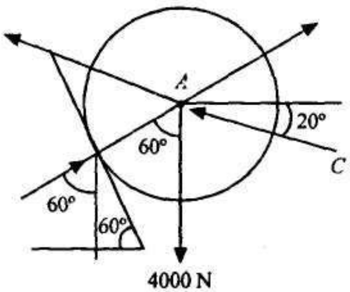



**Fig. 1.**

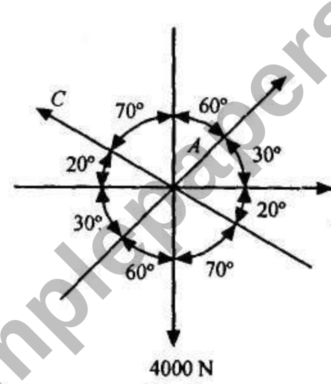



**Fig. 2.**

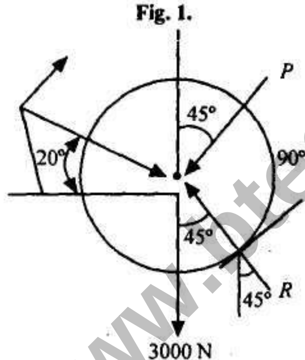



**Fig. 3.**

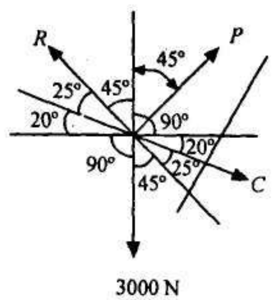



**Fig. 4.**

By the help of Lamis theorem from fig. 2.

$$\frac{C}{\sin(120^\circ)} = \frac{4000}{\sin(130^\circ)} = R$$

$\therefore$ $$C = 4522.06\text{ N}$$

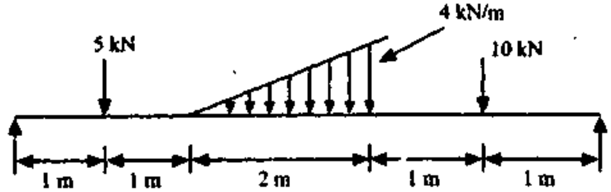
From fig. 3 summation of the forces parallel to the inclined plane (45° to horizontal) = 0

$$P + 3000\cos 45^\circ - C\cos 45^\circ = 0$$

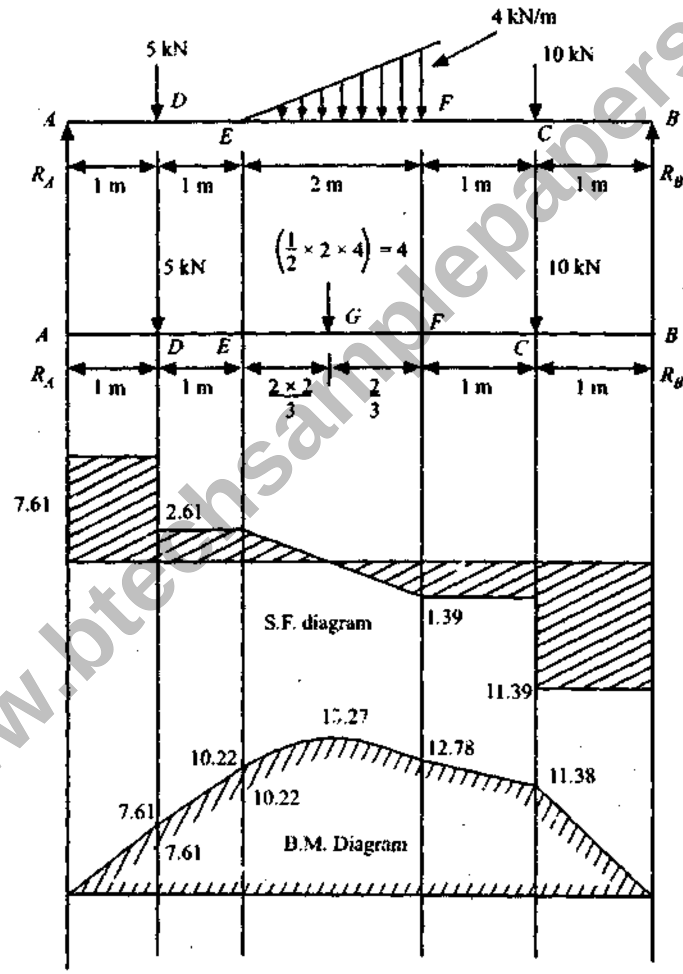
$\therefore$ $$P = 4522.06\cos 45^\circ - 3000\cos 45^\circ$$

$$= 1076.26\text{ N}$$

**Q. 2(b). Calculate the values of shear force and bending moments for the simple supported beam shown in Figure. Also draw the shear force nad bending moment diagrams.**




**Ans.**






By the help of equlibrium condition

$$\sum F_v = 0$$

$$R_A - 5 - 4 - 10 + R_B = 0$$

$$R_A + R_B = 19\,\text{kN}$$

$$\sum M_A = 0$$

$$R_A \times 0 - 5\times 1 - 4\times\left(2+\frac{4}{3}\right) - 10\times(5) + R_B \times 6 = 0$$

$\therefore$

$$R_B = 11.39\,\text{kN}$$
$$R_A = 7.61\,\text{kN}$$

S.F. between $AD = R_A = 7.61\,\text{kN}$

S.F. between $DE = 7.61 - 5 = 2.61\,\text{kN}$

S.F. between $GE = 2.61\,\text{kN}$

S.F. between $GF = 2.61 - 4$

$= -1.39\,\text{kN}$

S.F. between $FC = -1.39\,\text{kN}$

S.F. between $CB = -1.39 - 10$

$= -11.39\,\text{kN}$

S.F. at $B = -11.39 + 11.39 = 0$

B.m. at point $A = 0$

B.m. at point $B = 7.61\times 1 = 7.61\,\text{kN.m}$

B.m. at point $E = 7.61\times 2 - 5\times 1 = 10.22\,\text{kN.m}$

B.m. at point $G = 7.61\times\left(2+\frac{4}{3}\right) - 5\times\left(1+\frac{4}{3}\right)$

$= 13.7\,\text{kN.m}$

B.m. at point $F = 7.61\times(4) - 5\times 3 - 4\times\frac{2}{3}$

$= 12.78\,\text{kN.m}$

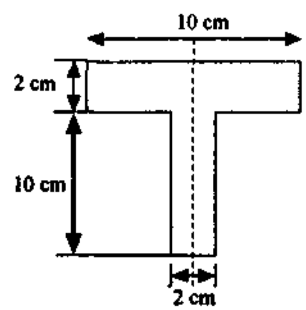
B.m. at point $C = 7.61\times 5 - 5\times 4 - 4\times\left(\frac{2}{3}+1\right)$

$= 11.38\,\text{kN.M}$

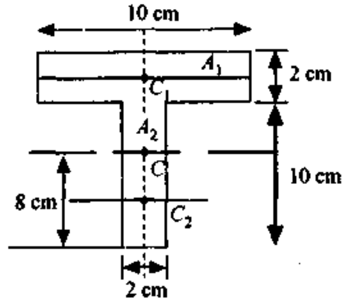
B.m. at point $B = 7.61\times 6 - 5\times 5 - 4\left(\frac{2}{3}+2\right) - 10\times 1$

$= 0\,\text{kN.M}$

**Q. 2(c). Determine the moment of inertia of T section about the horizontal and vertical axes, passing through the C.G. of the section as shown Fig.**



**Ans.**




| Figure | Area ($mm^2$) | $x$ co-ordinate of the centroid (mm) | $y$ co-ordinate of the centroid (mm) |
|---|---|---|---|
| Rectangle $A_1$<br>Rectangle $A_2$ | $A_1 = 2 \times 10 = 20\,mm^2$<br>$A_2 = 2 \times 10 = 20\,mm^2$ | $x_1 = 0$<br>$x_2 = 0$ | $y_1 = 11\,cm$<br>$y_2 = 5\,cm$ |

$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{20 \times 11 + 20 \times 5}{20 + 20} = 8\,mm$$

**Area $A_1$** : M.I. of area $A_1$ about its centroidal $x$-axis (that is the axis through $C_1$)

$$I_{x_1} = \frac{1}{12} \times 10 \times (2)^3$$

$$= 6.67\,mm^4$$

M.I. of are $A_1$ about the centroidal $x$ axis (that is the axis through $C_2$ )

$$(I'_x)_1 = (I_{x_1}) + A_1 d^2$$

$$= 6.67 + 20 \times (11 - 8)^2$$

$$= 186.67\,mm^4$$

**Area $A_2$** : M.I. of the area $A_2$ about its centroidal axis (that is the axis through $C_2$ )

$$(I_x)_2 = \frac{1}{12} \times 2 \times 10^3$$

$$= 166.67\,mm^4$$

M.I. of the area $A_2$ about the centroidal $x$ axis of the composite area

$$(I'_x)_2 = (I_x)_2 = A_2 d^2$$

$$= 166.67 + 20 \times (8 - 5)^2$$

$$= 346.67\,mm^4$$

Composite area $I'_x = (I'_{x_1}) + (I'_{x_2}) = (166.67 + 346.67)\,mm^4$

$$= 513.34\,mm^4$$

**Q. 2(d). A solid shaft is subjected to a maximum torque of 15 MN-cm. Determine the diameter of the shaft, if the allowable shear stress and the twist are limited to 1 kN/cm² and 1°, respectively for 210 cm length of shaft. $G$ = 8 MN/cm².**

**Ans.** $T = 15\,\text{MN-cm} = 15\times10^6\times10\,\text{N-mm} = 15\times10^7\ \text{N-mm} = 15\times10^3\ \text{kN-cm}$

Diameter based on shear stress :

$$\sigma_s = 1\,\text{kN/cm}^2 \qquad\qquad G = 8\,\text{MN/cm}^2$$

$$\frac{T}{J} = \frac{\sigma}{R} \qquad\qquad = 8\times10^3\ \text{kN/cm}^2$$

$$\therefore \quad \frac{15\times10^3}{\left(\frac{zD^4}{32}\right)} = \frac{1}{\frac{D}{2}} \qquad\qquad \therefore \quad \theta = 1 = \frac{z}{180}\ \text{red}$$

$$\therefore \quad \frac{15\times10^3\times32}{zD^2} = \frac{2}{D}$$

$$\therefore \quad D^3 = \frac{15\times10^3\times32}{2\times z}$$

$$\therefore \quad D = 42.43\,\text{cm}$$

Diameter based on angle of twist :

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\therefore \quad \frac{15\times10^3}{\frac{zD^4}{32}} = \frac{8\times10^3\times\frac{z}{180}}{210}$$

$$\therefore \quad \frac{15\times10^3\times32}{zD^4} = \frac{8\times10^3\times z}{180\times210}$$

$$D^4 = \frac{15\times10^3\times32\times180\times210}{8\times10^3\times z^2}$$

$$D = 21.89\,\text{cm}$$

So minimum diameter will be 21.89 cm.

**Q.2(e). The motion of a particle is given by $a = t^3 - 3t^2 + 5$, where $a$ is the acceleration in m/sec² and $t$ is the time in seconds. The velocity of the particle at $t$ = 1 sec is 6.25 m/sec, and the displacement is 8.30 meters. Calculate the displacement and the velocity at $t$ = 2 sec.**

**Ans.** Condition given are

at $t$ = 1 sec $v$ = 6.25 m/sec $x$ = 8.30 m

$$a = t^3 - 3t^2 + 5$$

$$a = \frac{dv}{dt}$$

$$= t^3 - 3t^2 + 5$$

$$dv = (t^3 - 3t^2 + 5)dt$$

$$v = \frac{t^4}{4} - \frac{3t^3}{3} + 5t + C_1$$

conditions are $t = 1, v = 6.25$

$\therefore$ $$6.25 = \frac{1}{4} - 1 + 5 + C_1$$

$$C_1 = 2$$

$\therefore$ $$v = \frac{t^4}{4} - t^3 + 5t + 2$$

$$v = \frac{dx}{dt} = \frac{t^4}{4} - t^3 + 5t + 2$$

$$dx = \left(\frac{t^4}{4} - t^3 + 5t + 2\right)dt$$

$$x = \frac{t^5}{5} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + C_2$$

Conditions are $t = 1, x = 8.30$

$\therefore$ $$8.30 = \frac{1}{5} - \frac{1}{4} + \frac{5}{2} + 2 + C_2$$

$$C_2 = 3.85$$

$\therefore$ $$x = \frac{t^5}{5} - \frac{t^4}{4} + \frac{5}{2}t^2 + 2t + 3.85$$

$\therefore$ Displacement $(x)$ at $t = 2$ we get

$$x = \frac{2^5}{5} - \frac{2^4}{4} + \frac{5}{2} \times 2^2 + 2 \times 2 + 3.85$$

$$= 20.25 \text{ m}$$

Velocity $(v)$ at $t = 2$ we get

$$v = \frac{2^4}{4} - 2^3 + 5 \times 2 + 2$$

$$= 8 \text{ m/sec.}$$

## Section-C

**Q. 3. Answer any two parts of the following :** **(2 × 5 = 10)**

**(a) State and prove Varignon's theorem.**

**(b) Derive an expression for the ratio of belt tensions in a flat belt drive.**

**(c) Explain briefly different types of friction.**

**Ans. (a) Varigons theorem :** It states that the algebric sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about point.

Consider force $P$ acting at a point $A$ and having components $P_1$ and $P_2$ in any two directions.

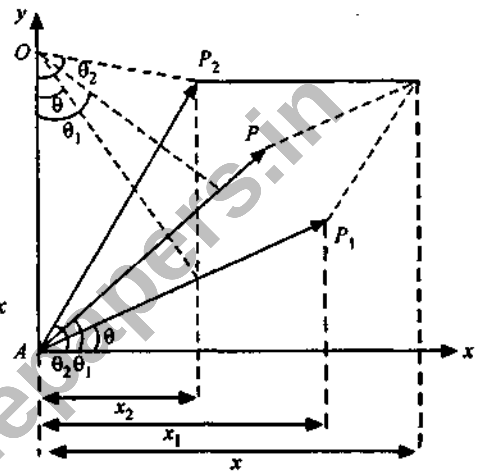
Let us consider any point '$O$' lying in the plane of the forces, as a moment centre.

Now, Moment of force $P$ about $O$ will be

$$Pd = P(OA\cos\theta)$$
$$= OA(P\cos\theta) = OA \times x \qquad \text{...(1)}$$

Moment of force $P_1$ about $O$

$$P_1 d_1 = P_1(OA\cos\theta_1)$$
$$= OA(P_1\cos\theta_1) = OA \times x_1 \qquad \text{...(2)}$$
$$P_2 d_2 = P_2(OA\cos\theta_2)$$
$$= OA(P_2\cos\theta_2) = OA \times x_2 \qquad \text{...(3)}$$




Adding equation (2) and (3)

$$P_1 d_1 + P_2 d_2 = OA(x_1 + x_2)$$

But $$x = x_1 + x_2$$

The sum of $x$ component of the forces $P_1$ and $P_2$ = $x$ component of the resultant $P$.

$\therefore$ $$OA \times x = P_1 d_1 + P_2 d_2$$

From equation (1)

$$OA \times x = Pd$$

$\therefore$ $$OA \times x = P_1 d_1 + P_2 d_2$$

$\therefore$ $$Pd = P_1 d_1 + P_2 d_2$$

**(b)** A driven pully rotating in clockwise direction is shown in figure.

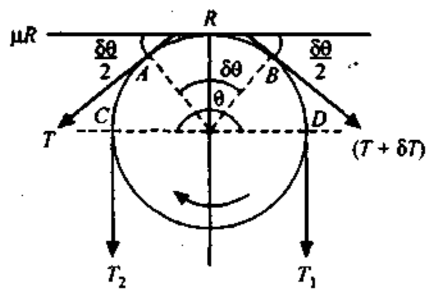
$T_1$ = Tension on tight side of belt

$T_2$ = Tension on slack side of belt

$\theta$ = angle of contact of belt with pulley

$\mu$ = coefficient of friction between the belt and pulley

Consider a small portion of belt $AB$ in contact with the pulley. $AB$ makes angle $\delta\theta$ at centre of pulley. Let $T$ and $(T + \delta T)$ be the tension at extremities $A$ and $B$.




The belt portion $AB$ is in equlibrium under the action of following forces.

Tension $T$ in belt at $A$

Tension $T + \delta T$ in belt at $B$

Normal reaction $R$ at $C$

Frictional forces $\mu R$

Now again solving the forces expression in tangential direction

$$\mu R + T\cos\left(\frac{\delta\theta}{2}\right) = (T + \delta T)\cos\frac{\delta\theta}{2}$$

Since $\delta\theta$ is very small $$\cos\left(\frac{\delta\theta}{2}\right) = 1$$

$$\mu R + T = T + \delta T$$
$$\mu R = \delta T$$

and resolving the force in radial direction

$$R = T\sin\left(\frac{\delta\theta}{2}\right) + (T + \delta T)\sin\left(\frac{\delta\theta}{2}\right)$$

Since $\delta\theta$ is very small, so

$$R = T\frac{\delta\theta}{2} + (T + \delta T)\frac{\delta\theta}{2} = T\delta\theta$$

$\therefore$ $$\mu(T\delta\theta) = \delta T$$

$$\frac{\delta T}{T} = \mu\delta\theta$$

on integration we get $$\int_{T_1}^{T_2}\frac{\delta T}{T} = \int_0^\theta \mu d\theta$$

$\therefore$ $$\ln\frac{T_1}{T_2} = \mu\theta$$

$\therefore$ $$\frac{T_1}{T_2} = \mu\theta$$

**(c) Friction is divided into two types.**

(i) Static friction

(ii) Dynamic friction

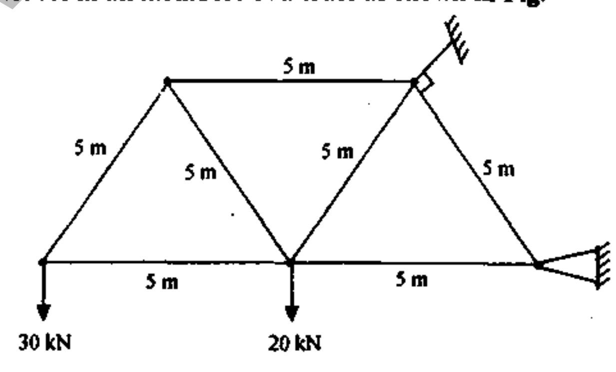
The forces of friction developed between the surface of contact of two bodies in the position of rest is known as static friction.

The force of friction developed between the surfaces of contact of two bodies in motion is known as dynamic friction.

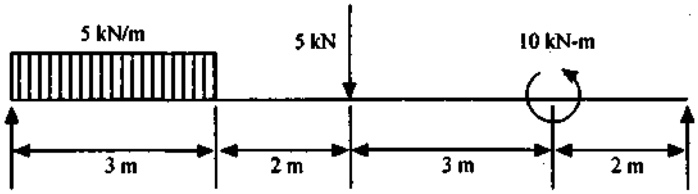
$I_f$ the surfaces is dry (not lubricated) then friction between surfaces is known as solid friction.

**Q. 4. Answer any one part of the following :** **10**

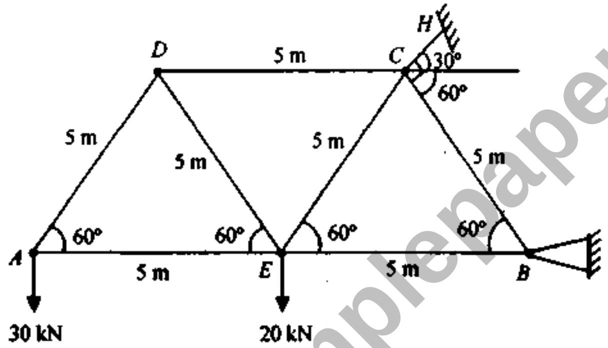
**(a) Find the axial forces in all members of a truss as shown in Fig.**



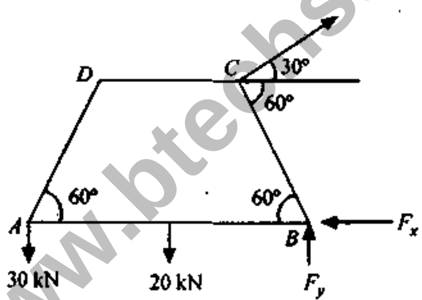
**(b) Draw the shear force and bending moment diagram for the beam loaded as shown in Fig.**

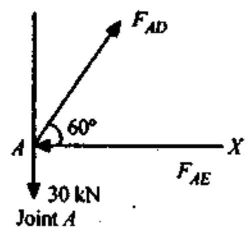



**Ans. (a)**




The inclination of CH is obtained from geometry.
Free body diagrams (a) and (b) are drawn to represent the forces. Name the members by Bow's method.




$$\sum F_x = 0 \qquad T\cos 30° - F_x = 0$$
$$\sum F_y = 0 \qquad T\sin 30° + F_y - 20 - 30 = 0$$
$$\sum M_B = 0 \qquad T\times 5 - 20\times 5 - 30\times 10 = 0$$

$\therefore$

$$T = 80\,\text{kN}$$
$$F_x = 80\cos 30° = 69.28\,\text{kN}$$
$$F_y = 50 - 80\sin 30° = 10\,\text{kN}$$

**For joint *A* :**

$\sum F_x = 0 \quad \therefore \qquad F_{AE} - F_{AD}\cos 60° = 0$

$\sum F_y = 0 \quad \therefore \qquad F_{AD}\sin 60° - 30 = 0$

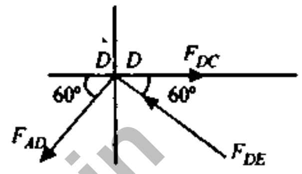
$\therefore$

$$F_{AD} = 34.64 \text{ kN}$$
$$F_{AE} = 17.32 \text{ kN}$$

**For joint $D$ :**

$\sum F_x = 0 \quad \therefore \quad F_{DE} \cos 60° - F_{DC} + F_{AD} \cos 60° = 0$

$\sum F_y = 0 \quad \therefore \quad F_{AD} \sin 60° - F_{DE} \sin 60° = 0$



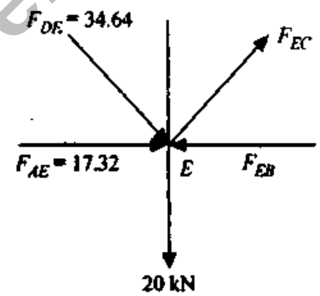

$$F_{DE} = \frac{34.64 \sin 60°}{\sin 60°} = 34.64 \text{ kN}$$

$$F_{DC} = F_{AD} \cos 60° + F_{DE} \cos 60°$$
$$= 34.64 \cos 60° + 34.64 \cos 60$$
$$= 34.64 \text{ kN}$$

**For Joint $E$ :**

$\sum F_x = 0 \quad \therefore \quad -F_{EB} + F_{AE} + F_{EC} \cos 60° + F_{DE} \cos 60° = 0$

$\sum F_y = 0 \quad \therefore \quad F_{DE} \sin 60° + 20 - F_{EC} \sin 60° = 0$



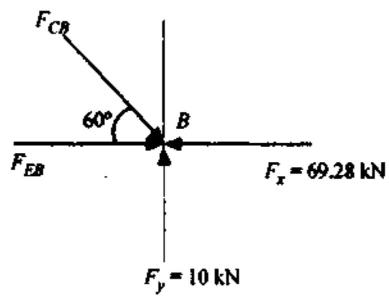

$$F_{EC} = \frac{34.64 \sin 60° + 20}{\sin 60°}$$
$$= 57.73 \text{ kN}$$

$$F_{EB} = F_{AE} + F_{EC} \cos 60° + F_{DE} \cos 60°$$
$$17.32 + 57.73 \cos 60 + 34.64 \cos 60$$
$$= 46.18 \text{ kN}$$

**For Joint B :**

$\sum F_x = 0 \quad \therefore \quad F_{EB} - F_x + F_{CB} \cos 60° = 0$

$\sum F_y = 0 \quad \therefore \quad F_y - F_{CB} \sin 60° = 0$



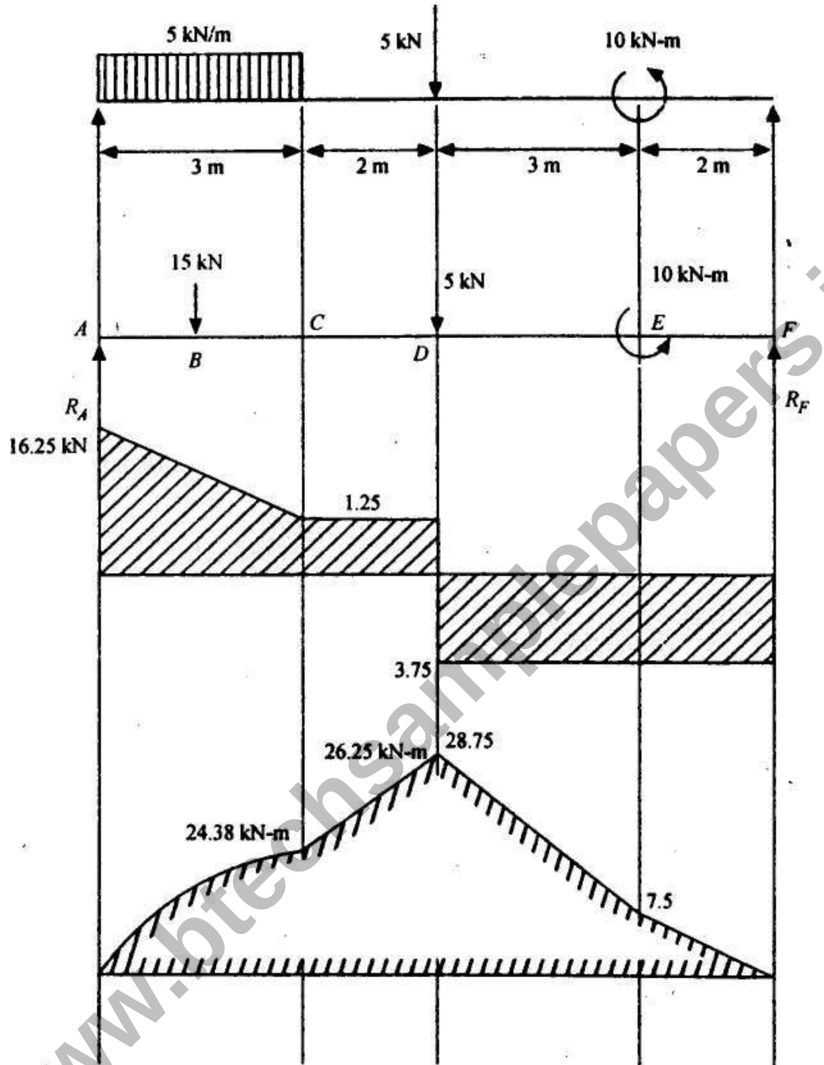

$\therefore$

$$F_{CB} = \frac{10}{\sin 60°} = 11.55 \text{ kN}$$

$$F_{EB} = F_x - F_{CB} \cos 60°$$
$$= 69.28 - 11.55 \cos 60°$$
$$= 63.51 \text{ kN}$$

| Forces | Magnitude | Nature |
|---|---|---|
| $F_{AD}$ | 34.64 kN | T |
| $F_{AE}$ | 17.32 kN | C |
| $F_{DE}$ | 34.64 kN | C |
| $F_{DC}$ | 34.64 kN | T |
| $F_{EC}$ | 57.73 kN | T |
| $F_{EB}$ | 46.18 kN | C |
| $F_{CB}$ | 11.55 kN | C |
| $F_{EB}$ | 63.51 kN | C |

**(b)**






By the help of equlibrium condition

$\sum F_y = 0 \;\therefore$ $\qquad R_A + R_F - 5 - 15 = 0$

$$R_A + R_F = 20\,\text{kN}$$

$\sum M_A = 0$ $\qquad 15\times 1.5 + 5\times 5 - 10 - R_F \times 10 = 0$

$$R_F = 3.75\,\text{kN}$$

$$R_A = 16.25\,\text{kN}$$

**S.F. Calculation :**

S.F. between $AB = 16.25\,\text{kN}$

S.F. between $BC = 16.25 - 15 = 1.25\,\text{kN}$

S.F. between $CD = 1.25 - 5 = -3.75\,\text{kN}$

S.F. between $DF = -3.75 + 3.75 = 0$

**B.M. Calculation :**

B.m. at point $A = 0$

B. m. at point $B = 16.25 \times 1.5 = 24.38$ kN-m

B.m. at point $C = 16.25 \times 3 - 15 \times 1.5 = 26.25$ kN-m

B.m. at point $D = 16.25 \times 5 - 15 \times 3.5 = 28.75$ kN-m

B.m. at point $E = 16.25 \times 8 - 15 \times 6.5 - 5 \times 3 - 10 = 7.5$ kN

B.m. at point $F = 16.25 \times 10 - 15 \times 8.5 - 5 \times 5 - 10 = 0$

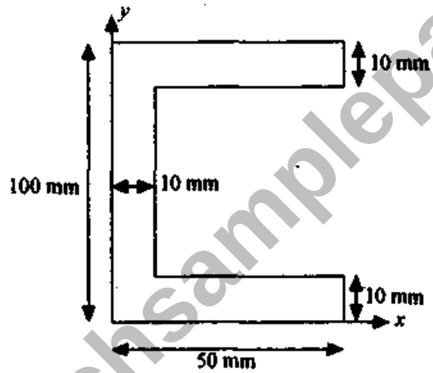
**Q. 5. Answer any two parts of the following : (2 × 5 = 10)**

**(a) Explain the following :**

**(i) Product of inertia**

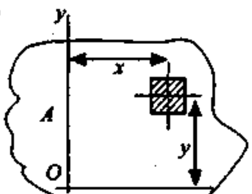
**(ii) Mass moment of inertia**

**(b) Locate the centroid of channel sectioo as shown in Figure.**

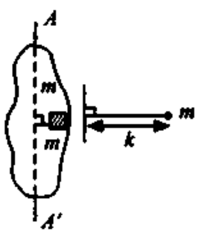



**(c) Determine the mass moment of inertia of a rectangular plate of size *axb* and thickness *t* about the centroidal axis.**

**Ans. (a) (i) Product of inertia :** Consider a plane figure of area $A$ in the $x-y$ plane. Divide this area into infinitesimal areas. The integral $I_{xy} \int xy\, dA$ obtained by multiplying each element $dA$ of the area $A$ by its coordinates $x$ and $y$ and the integration expanding over the entire area of the plane figure is called the product of inertia of the figure with respect to the $x$ and $y$ axes.




**(ii) Mass moment of inertia :** Consider a body of mass $m$. The moment of inertia of the body with respect the axis $AA'$ is defined by integral $I = \int r^2\, dm$ where $dm$ is the mass of an element of the body situated at a distance $r$ from the axis.




The axis $AA'$ and the integration is extended over the entire volume of the body.

The radious of $g$, ration $k$ of the body with respect to the axis $AA'$ is given by the relation

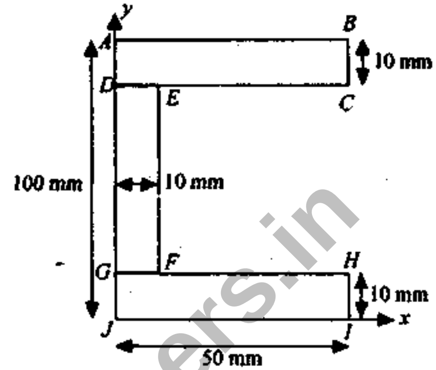
$$I = k^2 m$$

$$k = \sqrt{\frac{I}{m}}$$

(b) Area $A_1$ of figure $ABCD$ $50 \times 10 = 500\,\text{mm}^2$

Area $A_2$ of figure $DEFG$ $80 \times 10 = 800\,\text{mm}^2$

Area $A_3$ of figure $GHIJ$ $50 \times 10 = 500\,mm^2$




$$\therefore \quad \bar{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$= \frac{500 \times 25 + 800 \times 5 + 500 \times 25}{500 + 800 + 500}$$

$$= 16.11\,\text{mm}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$= \frac{500 \times 5 + 800 \times 50 + 500 \times 95}{500 + 800 + 500}$$

$$= 50\,\text{mm}$$

**(c) To find $I_{xx}$ :**

Consider an elemental strip of width $dy$ at a distance $y$ from $x$ axis

Mass of the element

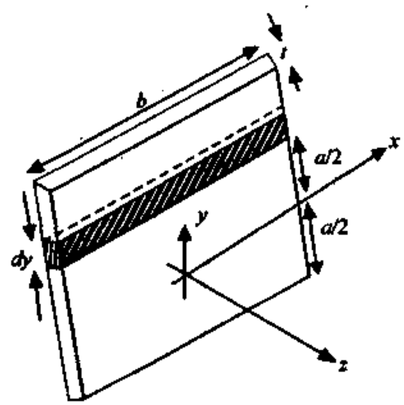
$$dm = \rho b \times t \times dy \qquad (\rho = \text{unit mass of material})$$

$$I_{xx} = \int_{-a/2}^{a/2} y^2\,dm$$

$$= \int_{-a/2}^{a/2} y^2 \rho bt\,dy$$

$$= \left[\frac{\rho bty^3}{3}\right]_{-a/2}^{a/2}$$

$$= \frac{\rho bta^3}{12}$$




But mass of the plate $m = \rho bta$

$$I_{xx} = \frac{Ma^2}{12}$$

**To find $I_{yy}$** : Taking an elemental strip parallel to axis,

$$I_{yy} = \frac{Mb^2}{12} \text{ (same as above)}$$

To find $I_{zz}$, consider an elemental area $dx \times dy$ and thickness

$$r^2 = x^2 + y^2$$

$$I_{zz} = \int r^2\,dm$$

$$= \int (x^2 + y^2) dm$$

$$= \int x^2 dm + \int y^2 dm$$

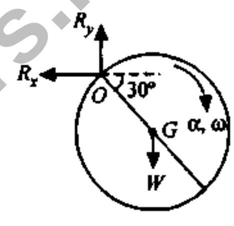
$$I_{zz} = I_{xx} + I_{yy}$$

$$= \frac{ma^2}{12} + \frac{mb^2}{12}$$

$$= \frac{m}{12}(a^2 + b^2)$$

**Q. 6. Answer any one part of the following :**

**(a) A train starts from rest and moves along a curved track of radius 600 m with uniform acceleration until it attains a velocity of 70 km/h at the end of third minute. Determine the tangential, normal and total acceleration of the train at the end of second minute.**




**(b) The cylinder shown in Figure is 70 cm in diameter and weighs 500 N. It is rotating about the fixed axis $O$ and has an angular velocity of 7 rad/s at the given instant. Using $D'$ Alembert's principle, find the horizontal and vertical components of the reaction at $O$.**

**Ans. (a)** At $t = 3$ sec, velocities of train $v = 72$ km/hour

$$= \frac{72 \times 10^3}{3600} \text{ m/sec} = 20 \text{ m/sec}$$

$$a_n = \frac{v^2}{r} = \frac{20 \times 20}{600} = 0.67 \text{ m/sec}^2 \qquad \text{(Normal acceleration component)}$$

$$a_t = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{20-0}{3} = 6.67 \text{ m/sec}^2$$

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(r\cos \omega t) = -\omega r \sin \omega t$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(r \sin \omega t) = \omega r \cos \omega t$$

$t = 2$ sec, $\omega = \frac{v}{r}$

$$v = u + at$$

$\therefore$

$$20 = 0 + a \times 3$$

$$a = \frac{20}{3} = 6.67 \text{ m/sec}^2$$

at $t = 2$

$$v = u + at$$

$$v = 0 + 6.67 \times 2$$

$$= 13.34 \text{ m/sec}$$

$a_t$ = tangential acceleration at $t = 2$

$$a_t = \frac{dv}{dt} = \frac{13.34}{2} = 6.67 \text{ m/sec}^2$$

$a_n$ = normal acceleration at $t = 2$

$$a_n = \frac{v^2}{r} = \frac{(13.34)^2}{600} = 0.29\text{ m/sec}^2$$

$$\omega = \frac{v}{r} = \frac{13.34}{600} = 0.02\text{ radium/sec}$$

$t = 2\text{ sec}, \omega = 0.02\text{ rad/sec} = 0.02 \times \frac{180}{\pi}$ degrees/sec $r = 600\text{ m}$

$$v_x = -0.02 \times 600 \sin\left(0.02 \times 2 \times \frac{180}{\pi}\right)$$

$$= 0.48\text{ m/sec}$$

$$v_y = 0.02 \times 600 \cos\left(0.02 \times 2 \times \frac{180}{\pi}\right)$$

$$= 11.99\text{ m/sec}$$

$$a_x = -\omega^2 r \cos \omega t$$

$$= -(0.02)^2 \times 600 \times \cos\left(0.02 \times 2 \times \frac{180}{\pi}\right)$$

$$= -0.24\text{ m/sec}^2$$

$$a_y = -\omega^2 r \sin \omega t$$

$$= -(0.02)^2 \times 600 \times \sin\left(0.02 \times 2 \times \frac{180}{\pi}\right)$$

$$= -0.0096\text{ m/sec}^2$$

Total acceleration $(a) = \sqrt{a_x^2 + a_y^2}$

$$= \sqrt{(0.24)^2 + (0.0096)^2}$$

$$= 0.24\text{ m/sec}^2$$

**Ans. (b)** Diameter of cylinder $D = 70\text{ cm}$

moment of inertia of the body

$$I = \frac{1}{2}Mr^2 = \frac{1}{2} \times \frac{500}{9.8} \times \left(\frac{35}{100}\right)^2 = 4.25$$

$\therefore r$ = radious of the cylinder

$$= 35\text{ cm} = -35\text{ m}$$

Moment about $G$

$$R_y \times r\cos 30° - R_x \times r\sin\theta - I\alpha = 0$$

By equilibrium condition

$$W = R_y$$

$\therefore$ $R_y = 500\text{ N}$

$R_x$ and $R_y$ are the horizontal and vertical component.

$$500 \times 0.35 \cos 30° - R_x \times 0.35 \sin 30° - 4.25 \times 7 = 0$$

$$R_x = 121.8\text{ N}$$

**Q. 7. Answer any two of the following :**

**(a) A 300 mm deep rectangular beam is simply supported over a span of 6 m. What uniformly distributed load per meter the beam can carry if bending stress is not to exceed 110 N/mm². Take $I = 8.5 \times 10^6$ mm².**

**(b) A rectangular bar of uniform cross-section 4 cm × 2.5 cm and of length 2.2 m is hanging vertically from a rigid support. It is subjected to axial tensile loading of 10 kN. If density of steel is 8000 kg/m³ and $E = 200$ GN/m², find the maximum stress and the elongation of the bar.**

**(c) Derive the torsion formula $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$.**

**Ans. (a)**

$$\frac{M}{I} = \frac{f_b}{y}$$

$f_b = 110$ N/mm², $I = 8.5 \times 10^6$ mm², $y = \frac{300}{2}$ mm

$$\therefore \quad M = \frac{f_b I}{y}$$

$$= \frac{110 \times 8.5 \times 10^6}{150}$$

$$= 6.23 \times 10^6 \text{ N-mm}$$

For U.D.L. maximum bending moment is $\frac{\omega L^2}{8} = 6.23 \times 10^6$

$$L = 6\text{m} = 6 \times 10^3 \text{ mm}$$

$$\therefore \quad \frac{\omega \times 6 \times 10^3}{8} = 6.23 \times 10^6$$

$$\therefore \quad \omega = \frac{6.23 \times 10^6 \times 8}{6 \times 10^3}$$

$$= 8.31 \times 10^3 \text{ N/mm}$$

$\omega = 8.31$ kN/mm is the value of uniformless distributed load.

**(b)**

$$A = (4 \times 2.5) \text{ cm}^n$$

$$= 10 \text{ cm}^n = 10 \times (10^{-2})^2 \text{ m}^n$$

$$= 10 \times 10^{-4} \text{ m}^n$$

$$L = 2.2 \text{ m}$$

Tensile stress $(\sigma_t) = \frac{F}{A} = \frac{10 \times 10^3}{10 \times 10^{-4}}$

$$= \frac{10 \times 10^3 \times 10^4}{10} = 10^7 \text{ N/m}^2$$

Self weight of the material $= 8000 \times (4 \times 2.5) \times 2.2 \times 10^{-4}$ kg

$$= 17.6 \text{ kg} \times g = 172.48 \text{ N}$$

$\therefore$ Maximum tensile stress $= \sigma_{max}$

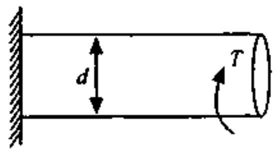
$$= \frac{17248 + 10\times10^3}{(10\times10^{-4})} \text{ N/m}^2$$

$$E\text{ longation } (\Delta) = \frac{F\times L^2}{2E}$$

$$= \frac{(17248 + 10\times10^3)\times(22)^2}{2\times200\times10^9} \text{ m}$$

$$= 123\times10^{-7} \text{ m}$$

**(c)** A circular shaft is subjected to pure torque '*T*' about its polar axis, shear stresses set up in directions perpendicular to the radius on all transverse sections. The complementry shear stress on longitudinal plane will cause a distortion of filaments which are originally in the longitudinal direction.





A generator on the surface of the shaft denoted by *AB*, deforms into the configuration *AC* after tension has occured. The angle between these configuration is denoted by *x*. By definition, the shearing strain on the surface of the shaft is

$$\gamma = \tan\alpha \simeq \alpha$$

$$\alpha = \frac{BC}{L} = \frac{r.\theta}{L}$$

$$\therefore \qquad \gamma = \frac{r\cdot\theta}{L}$$

But since a diameter of the shaft before loading is assumed to remain a diameter after torsion, the shearing strain at a general distance ρ from the centre of the shaft is given by $\gamma_p = \frac{\rho\cdot\theta}{L}$.

Sum of the moments of the distributed shearing forces over the entire circular cross section = Applied twisting moment

$$\Rightarrow \qquad \int_0^r \tau_\rho \rho.\, da = T$$

$$= \frac{\tau_\rho}{\rho} = \frac{\tau}{r}$$

$$= \text{constant}$$

Consequently, dividing and multiplying in ρ in right hand side of equation

$$T = \int_0^r \frac{\tau_\rho}{\rho}(\rho^2)da$$

$$T = \frac{\tau_\rho}{\rho}\int_0^r \rho^2 \cdot da$$

$$T = \frac{\tau_\rho}{\rho}\int_0^r \rho^2\, 2z\rho d\rho$$

$$= \frac{\tau_\rho}{\rho} \int_0^r 2z\rho^3 . d\rho$$

$$= \frac{\tau}{r} .2z \left| \frac{\rho^4}{4} \right|_0^r$$

$$= \frac{\tau}{r} \frac{\pi}{2} r^4$$

$$= \frac{z}{16} \tau . d^3$$

Since the ratio $\frac{\tau_\rho}{\rho}$ is a constant and the quantities $\int_0^r \rho^2 . da$ is called polar moment of inertia by definition, denoted by $J$ hence,

$$T = \frac{\tau_\rho . J}{\rho}$$

Now for outermost fiber where $\rho = r$

$$\gamma_r = \frac{r.\theta}{2}$$

and $$\tau = \frac{T.r}{J} \text{ or } \frac{T}{J} = \frac{\tau}{r}$$

By definition of modulus of elasticity $G = \frac{\tau}{J}$

$$G = \frac{\tau}{\gamma} = \frac{T.r}{J} / \frac{r.\theta}{L} = \frac{T}{J} \frac{L}{\theta}$$

$$G = \frac{T.L}{J.\theta}$$

$$J = \frac{T.L}{G.\theta}$$

or $$\frac{T}{J} = \frac{G.\theta}{L} = \frac{\tau}{r}$$